Sensitivity analysis with GP emulator for cardiovascular modelling

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Quantifying uncertainty in multiscale models for biomedical applications

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INSIGNEO
Institute for *in silico* Medicine

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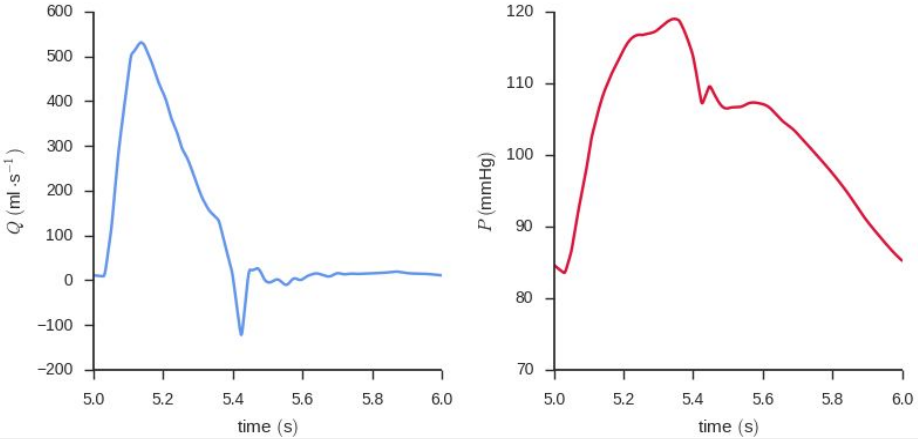
The University Of Sheffield.

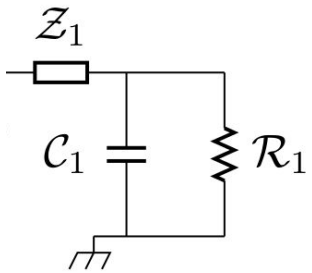
Outline

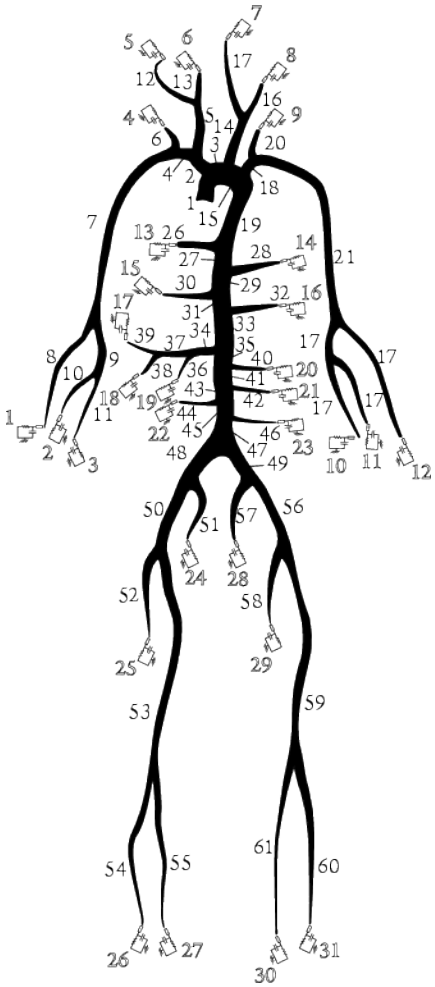
- 1D vascular models
- Sensitivity analysis
- Scalability issue
- Gaussian process
- Cerebral vasospasm
- Results
- Conclusions

1D cardiovascular models

- Simple mechanical description:
  - Length, radius, Young's modulus
  - Network topology
- Lumped parameters outlet BCs
- Time varying inlet flow function

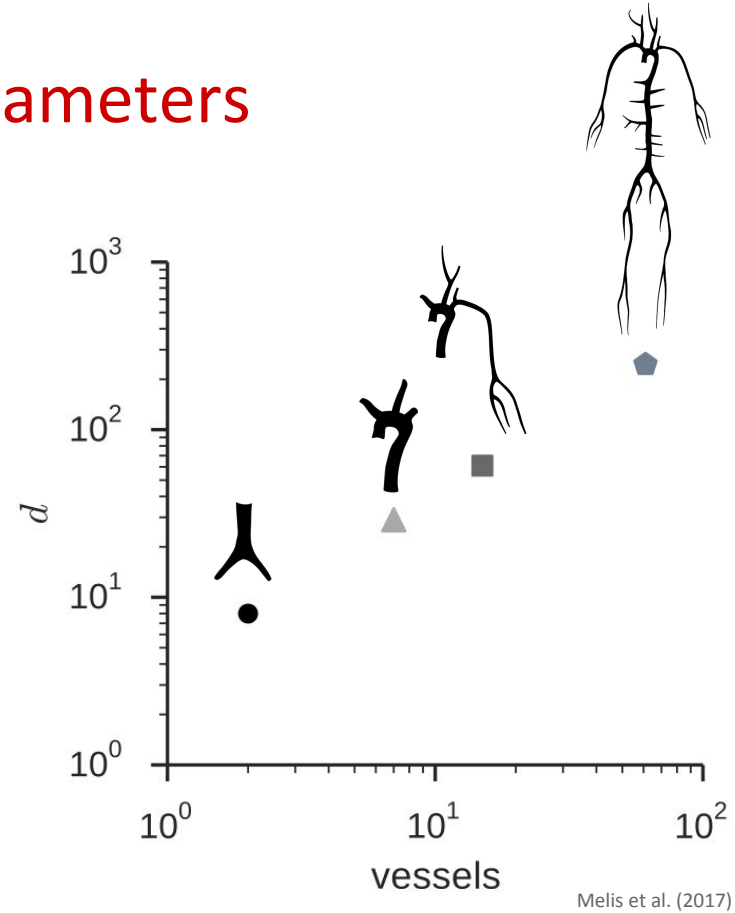






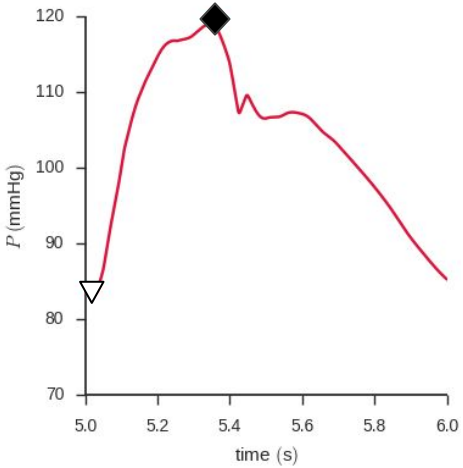
1D cardiovascular models: parameters

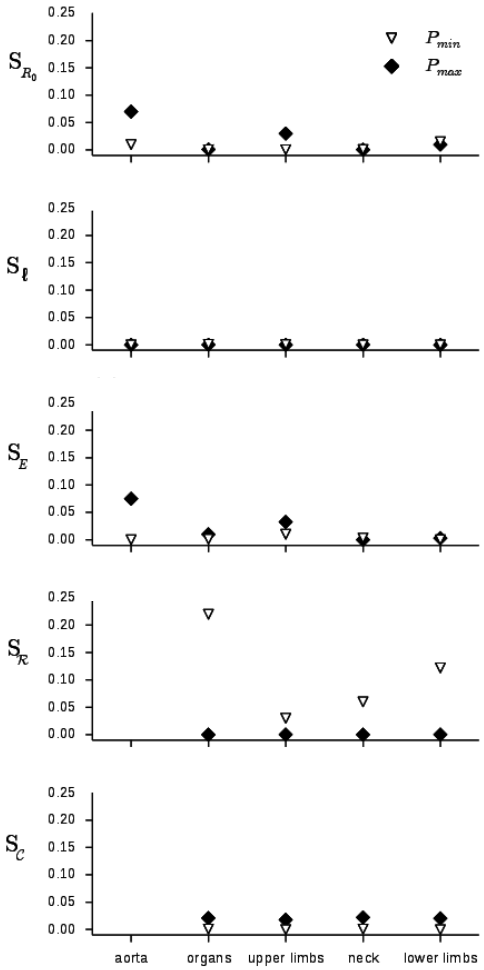
- *d* number of parameters
- Linear increase with number of vessels
- Ideally, all the parameters should be directly measured on the subject
- Sensitivity analysis:
  - Parameter ranking
  - Parameter fixing

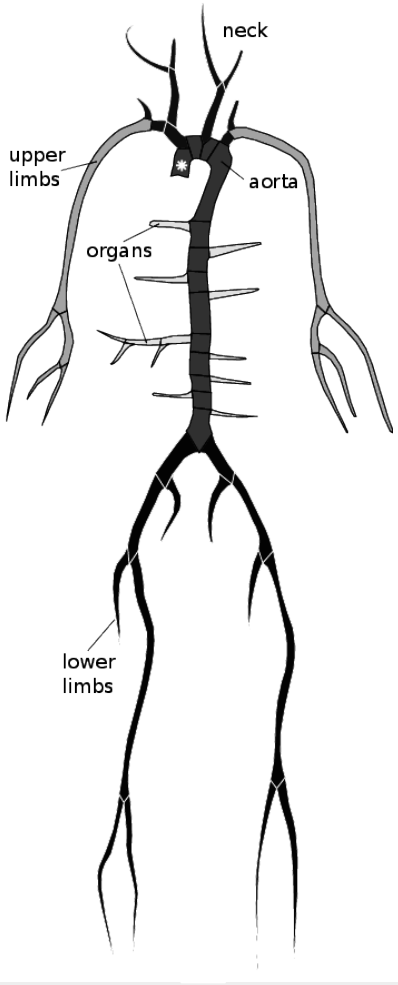


Melis et al. (2017)

Sensitivity analysis

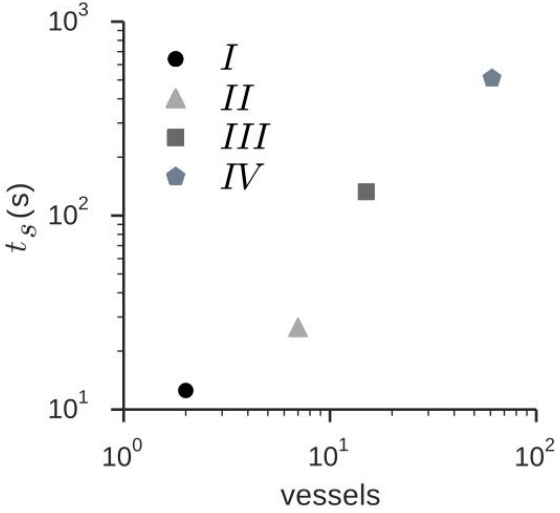


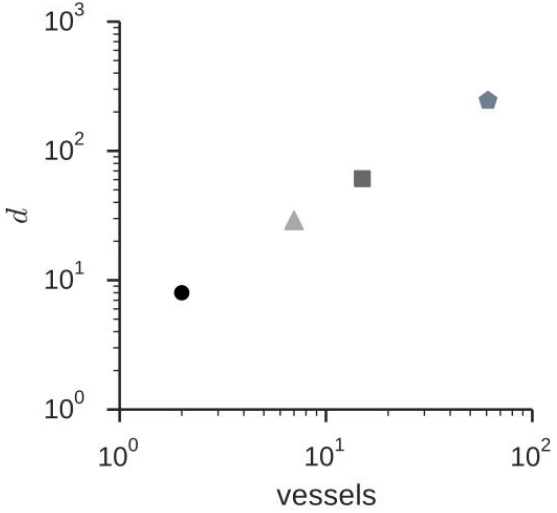


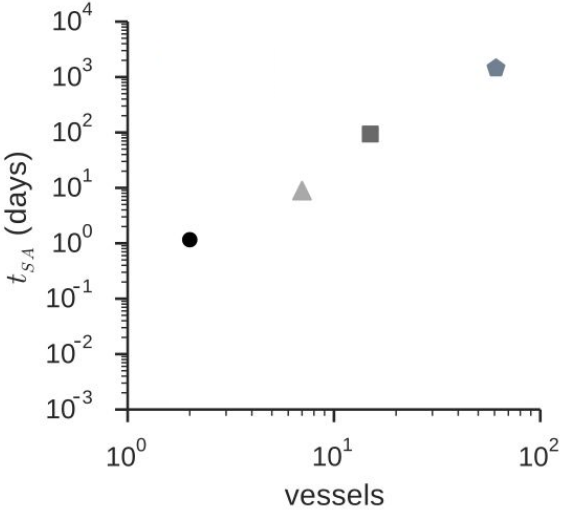


Melis, A., Clayton, R. H., and Marzo, A. (2017) **Bayesian sensitivity analysis of a 1D vascular model with Gaussian process emulators**. Int. J. Numer. Meth. Biomed. Engng., doi: 10.1002/cnm.2882.

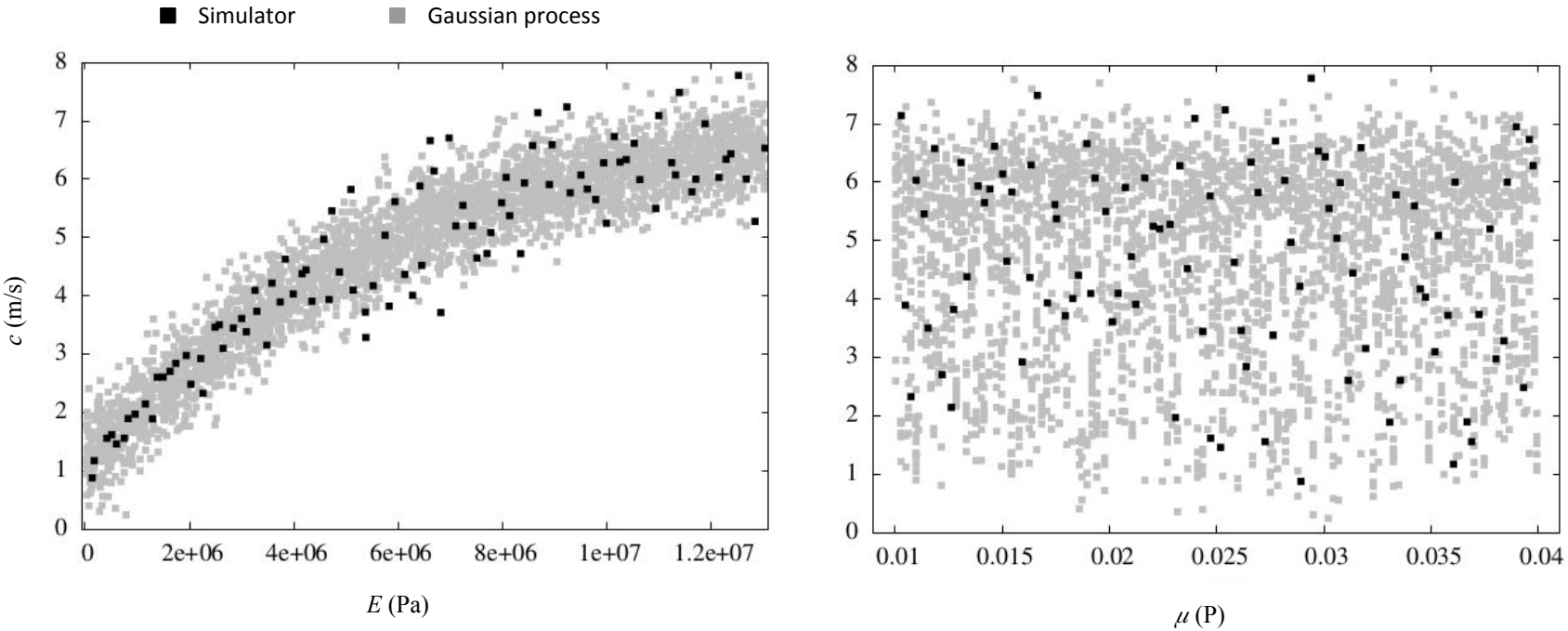
Scalability







Gaussian process predictions

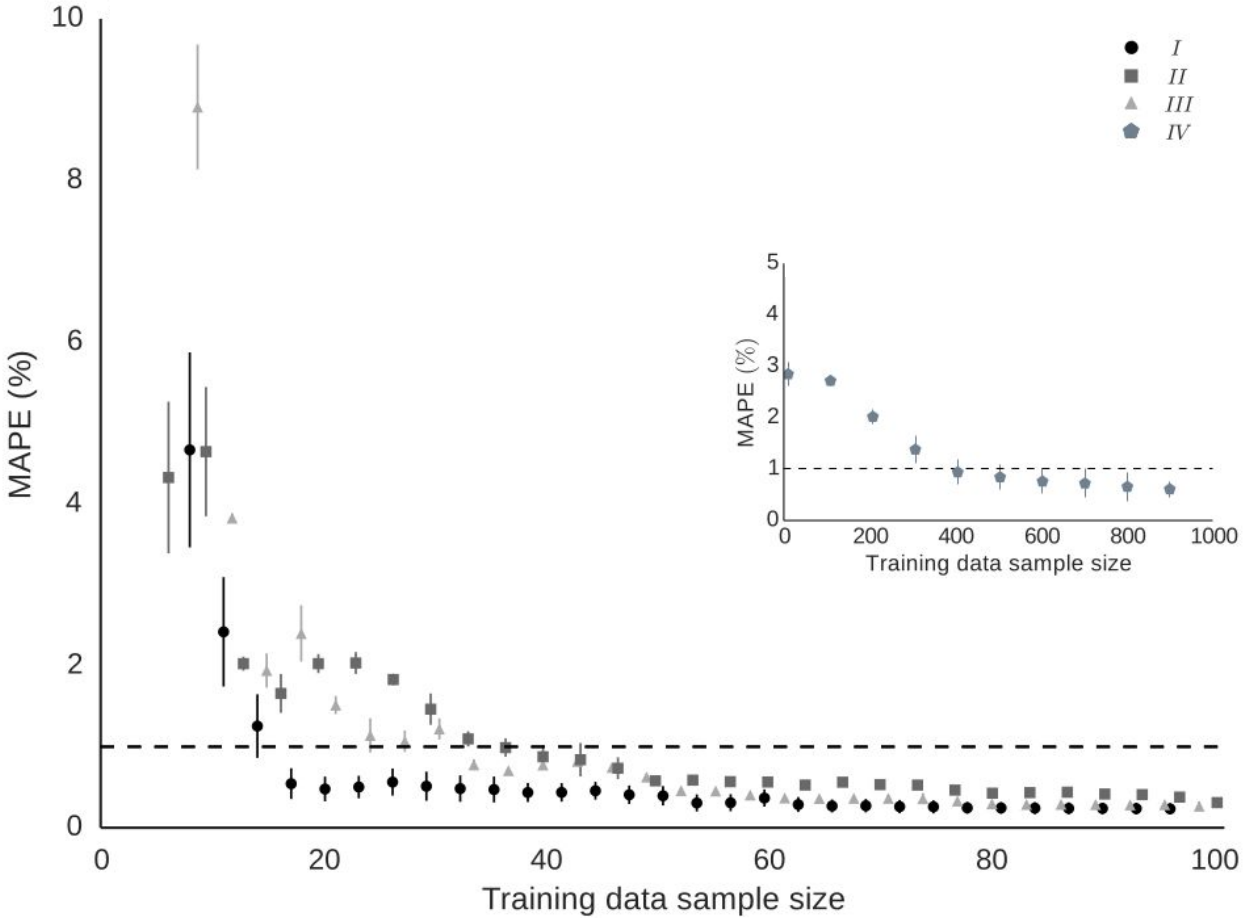


Melis et al. (2015) CMBE2015 proceedings

Prediction error

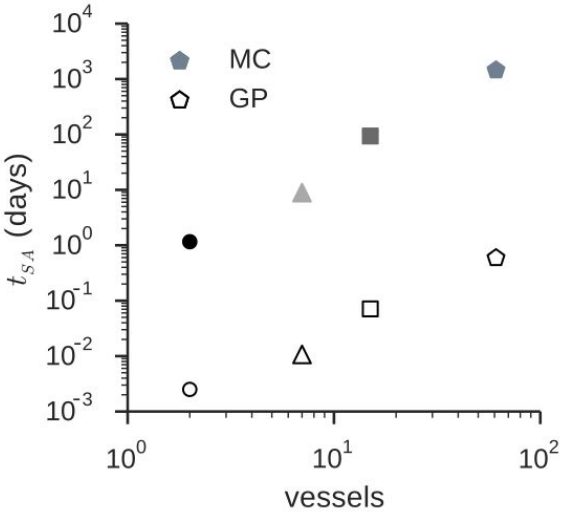
	d	N_{GP}	N_{MC}
I	8	17	8000
II	29	34	29000
III	61	47	61000
IV	245	500	245000

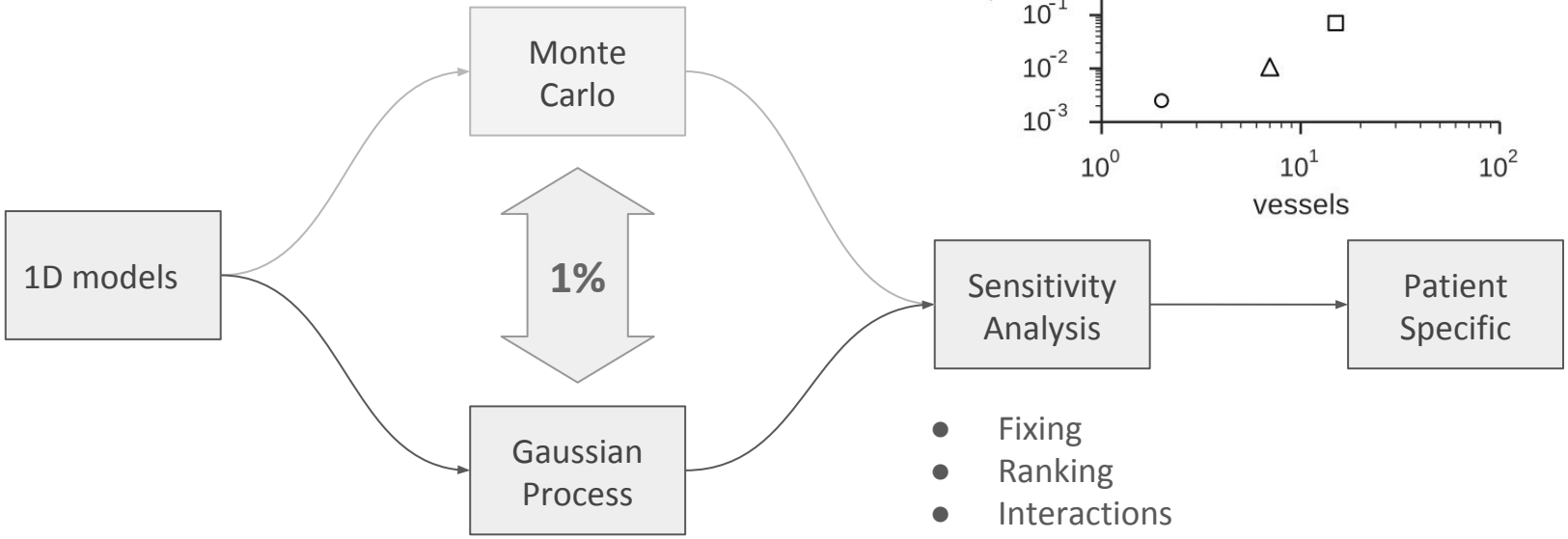
$$\mathcal{O}(d \times 10^3) \rightarrow \mathcal{O}(d)$$



GP for sensitivity analysis

$\mathcal{O}(d \times 10^3)$

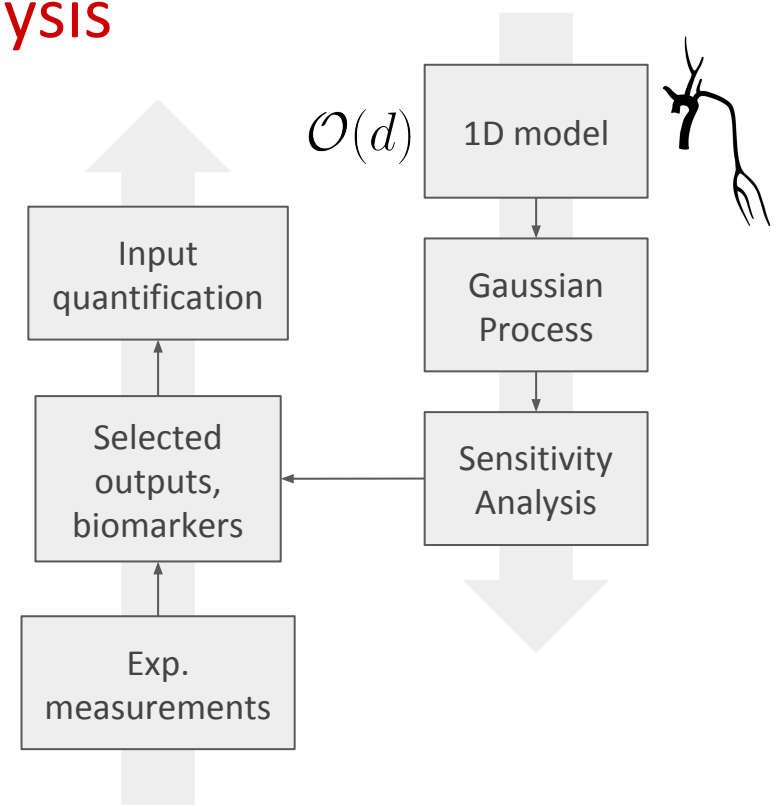




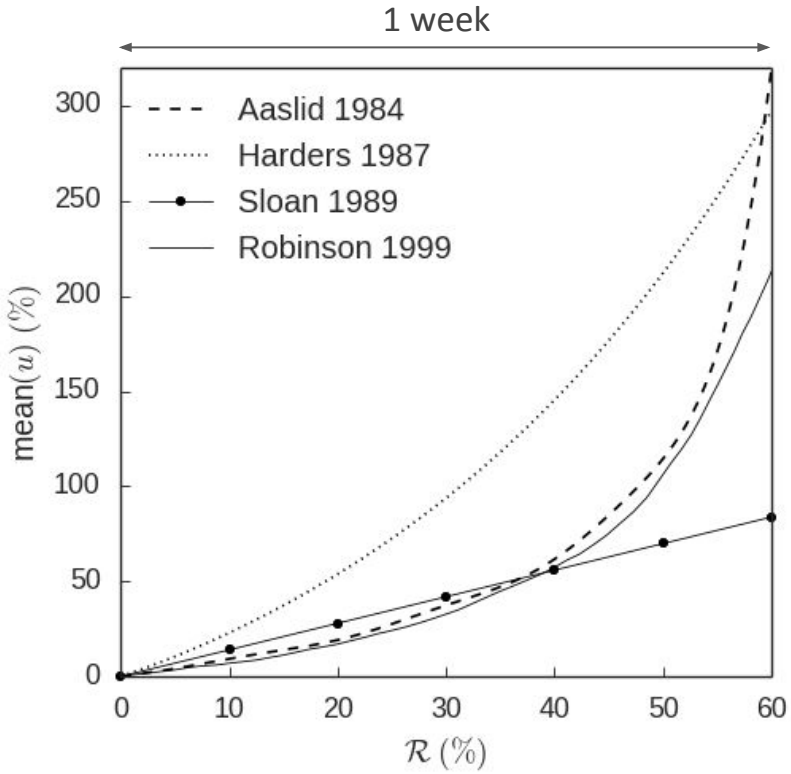
$\mathcal{O}(d)$

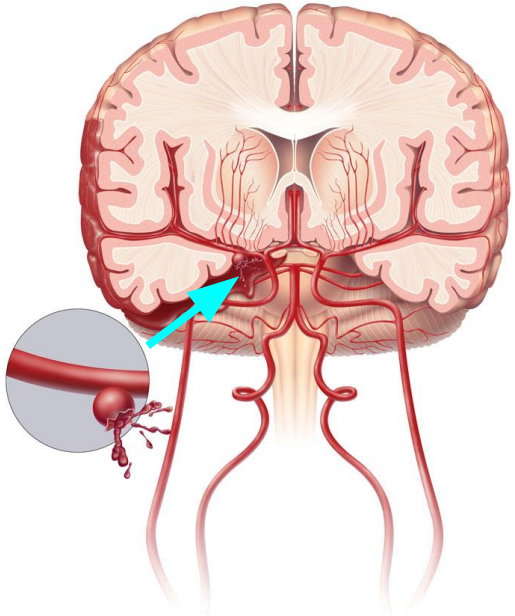
Biomarkers from sensitivity analysis

If inputs are unknown and we notice a change in one of the outputs, can we identify the input more likely to cause this change?



Cerebral vasospasm (CVS)





strokecenter.org

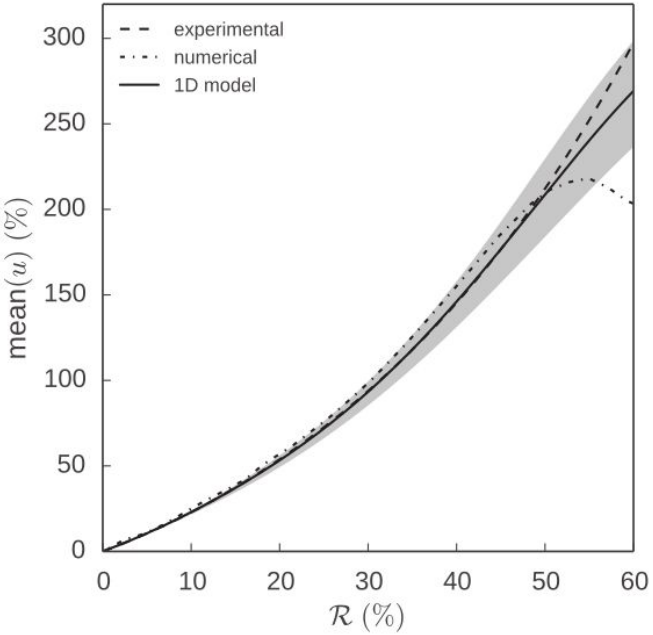
Research questions

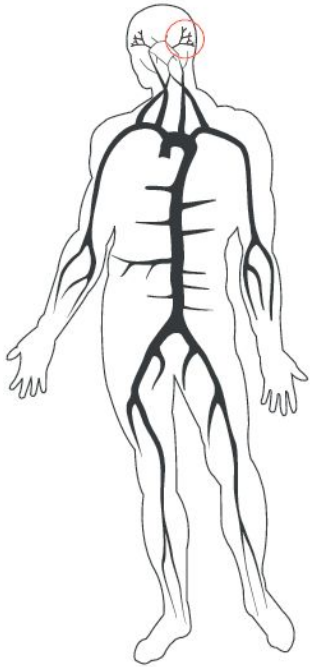
- Are there multiple types of CVS?
- Are there more effective biomechanical metrics (biomarkers) for CVS detection?

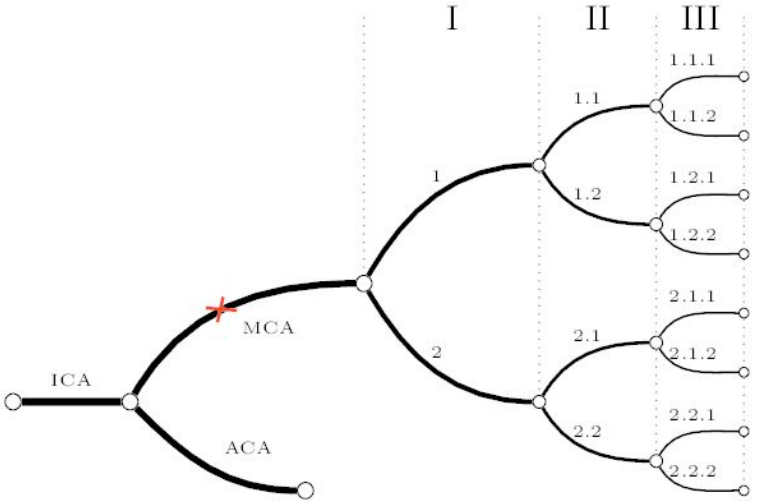
  More effective = more sensitive to changes in lumen radius only

- Can we classify the type of CVS upon biomarker readings?

Cerebral vasospasm: 1D model







Melis et al. (2017) IEEE Trans. Biom. Eng. [in review]

Cerebral vasospasm: sensitivity analysis

- 17 vessels
- 5 parameters
- +/- 50% variation
- 27 features measured on MCA waveforms
- 50 model runs for GP training
- SA dataset generated through GP regression

Sensitivity analysis results

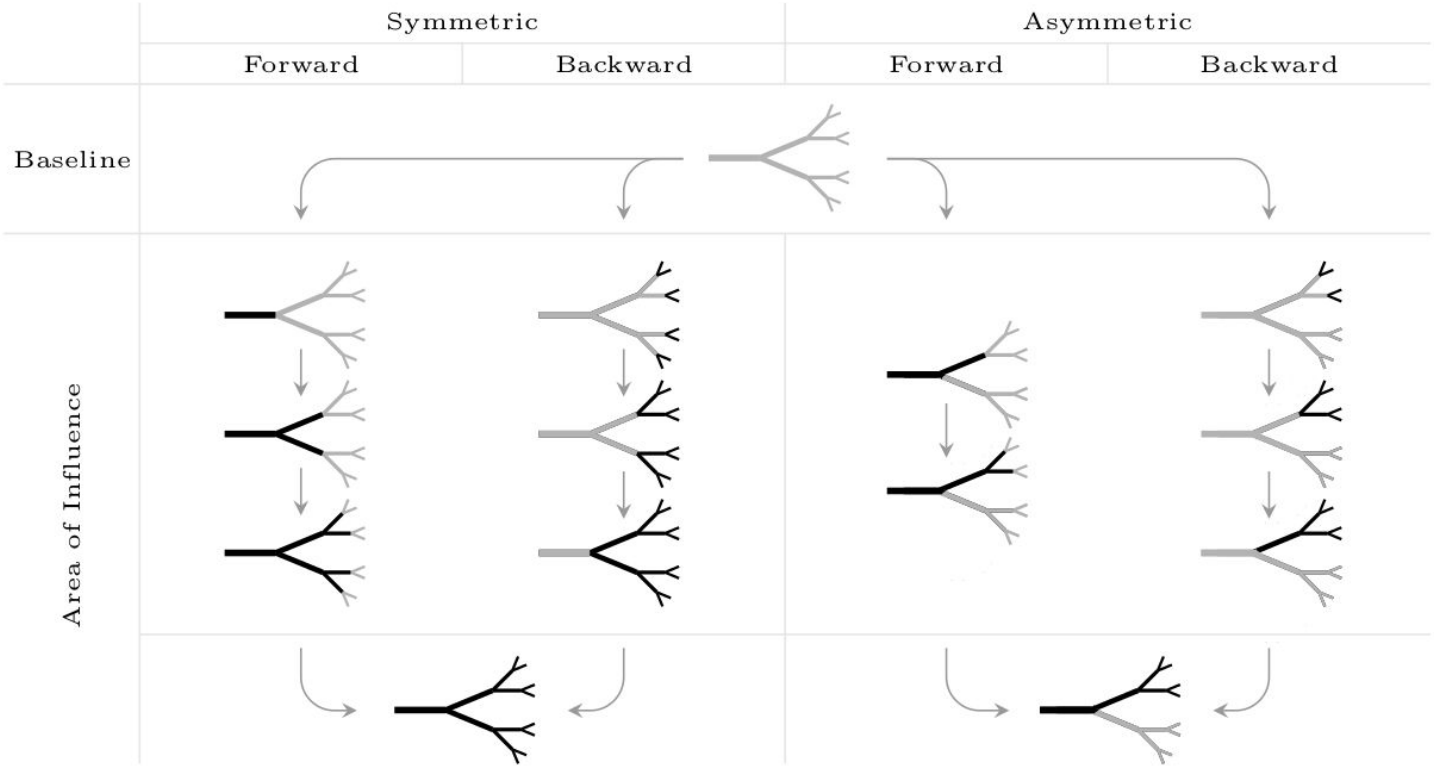
Include only

- T > 90%
- H < 5%

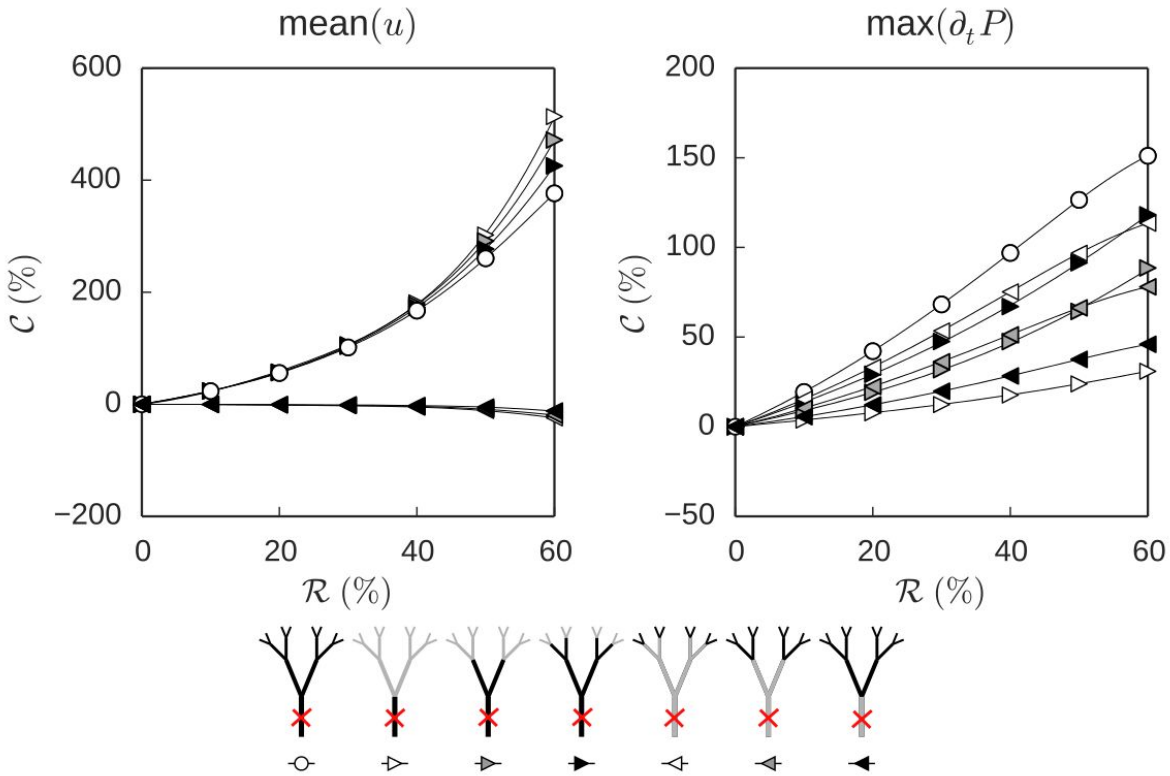
CV biomarker	$\mathbf{T}_{R_0}$	$\mathbf{H}_{R_0}$
$\mathrm{mean}(u)$	97.43	0.82
$\max(u)$	97.28	0.84
$\min(u)$	97.23	0.93
$\max(\partial_t u)$	96.83	0.90
$\min(\partial_{tt} P)$	93.76	3.62
$\max(\partial_{tt} P)$	93.05	3.35
$\min(\partial_{tt} u)$	92.98	1.16
$\max(\partial_t P)$	92.52	3.18

	$\mathbf{T}_i$					$\mathbf{H}_i$				
$\min(P)$	1	1	1	99	0	0	1	1	3	0
$\min(\partial_t P)$	90	7	6	0	2	5	3	5	0	1
$\min(\partial_{tt} P)$	94	6	4	0	0	4	3	3	0	0
$\min(Q)$	71	22	36	7	6	66	24	23	5	2
$\min(\partial_t Q)$	88	5	12	0	4	10	1	6	0	1
$\min(\partial_{tt} Q)$	73	7	25	1	5	8	4	9	0	0
$\min(u)$	97	0	2	0	0	1	1	1	0	0
$\min(\partial_t u)$	89	1	6	0	6	4	2	5	0	1
$\min(\partial_{tt} u)$	93	1	5	0	1	1	2	3	0	0
$\mathrm{mean}(P)$	3	0	1	98	0	0	0	0	3	0
$\mathrm{mean}(\partial_t P)$	50	33	46	9	5	32	23	33	1	3
$\mathrm{mean}(\partial_{tt} P)$	31	50	41	32	0	31	49	45	3	0
$\mathrm{mean}(Q)$	81	9	17	14	4	60	36	52	14	2
$\mathrm{mean}(\partial_t Q)$	59	54	54	0	1	46	43	43	1	0
$\mathrm{mean}(\partial_{tt} Q)$	65	35	65	0	1	54	27	46	1	1
$\mathrm{mean}(u)$	97	0	2	0	0	1	1	1	0	0
$\mathrm{mean}(\partial_t u)$	92	4	13	2	0	8	2	11	0	0
$\mathrm{mean}(\partial_{tt} u)$	88	16	35	0	2	29	10	28	0	2
$\max(P)$	18	1	2	81	0	1	1	1	3	0
$\max(\partial_t P)$	93	7	4	0	1	3	3	4	0	0
$\max(\partial_{tt} P)$	93	6	4	0	1	3	3	4	0	0
$\max(Q)$	92	3	16	4	1	41	5	24	4	1
$\max(\partial_t Q)$	86	2	17	3	2	28	1	15	2	1
$\max(\partial_{tt} Q)$	83	8	13	0	6	9	4	7	0	0
$\max(u)$	97	0	2	0	0	1	1	1	0	0
$\max(\partial_t u)$	97	1	2	0	0	1	1	1	0	0
$\max(\partial_{tt} u)$	88	3	8	0	5	3	3	4	0	0
	R_0	E	ℓ	R_p	C_p	R_0	E	ℓ	R_p	C_p

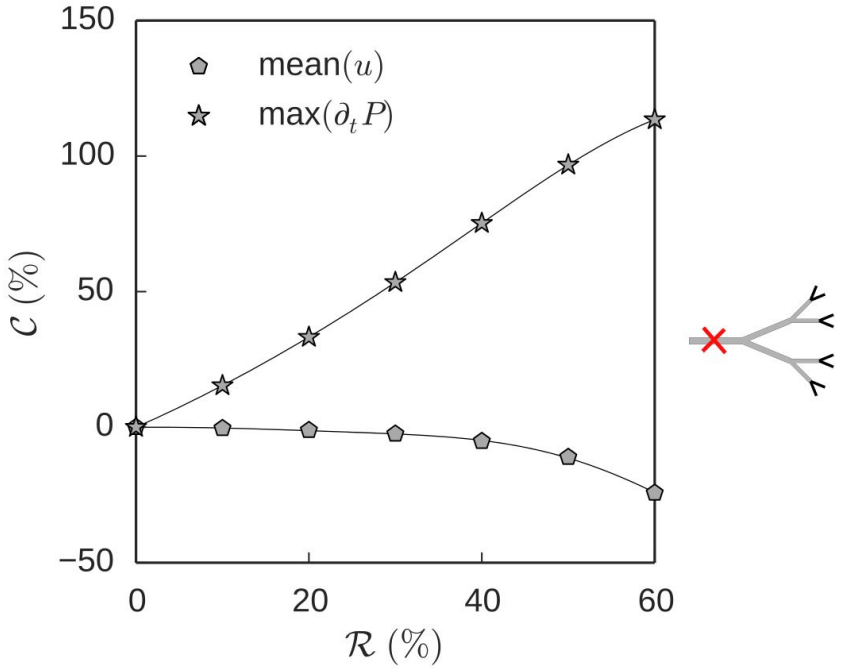
Cerebral vasospasm simulations



Biomarkers


mean(u)
600
400
200
0
−200
$\mathcal{C}$ (%)
0
20
40
60
$\mathcal{R}$ (%)
max$(\partial_t P)$
200
150
100
50
0
−50
$\mathcal{C}$ (%)
0
20
40
60
$\mathcal{R}$ (%)

Biomarkers comparison



Conclusions

- Sensitivity analysis:
  - Reduce model dimensionality (ranking and fixing)
  - Measurements *reverse engineering*
  - Requires large amount of data
- Gaussian process:
  - Trained on few runs of the deterministic model (computationally cheap)
  - Generates the dataset for SA computation
- Cerebral vasospasm:
  - Identified CVS types
  - Identified biomarker sensitive to CVS only (through SA)
  - CVS types classification based on biomarker
  - ➢ Basis for further experimental validation

Thank you

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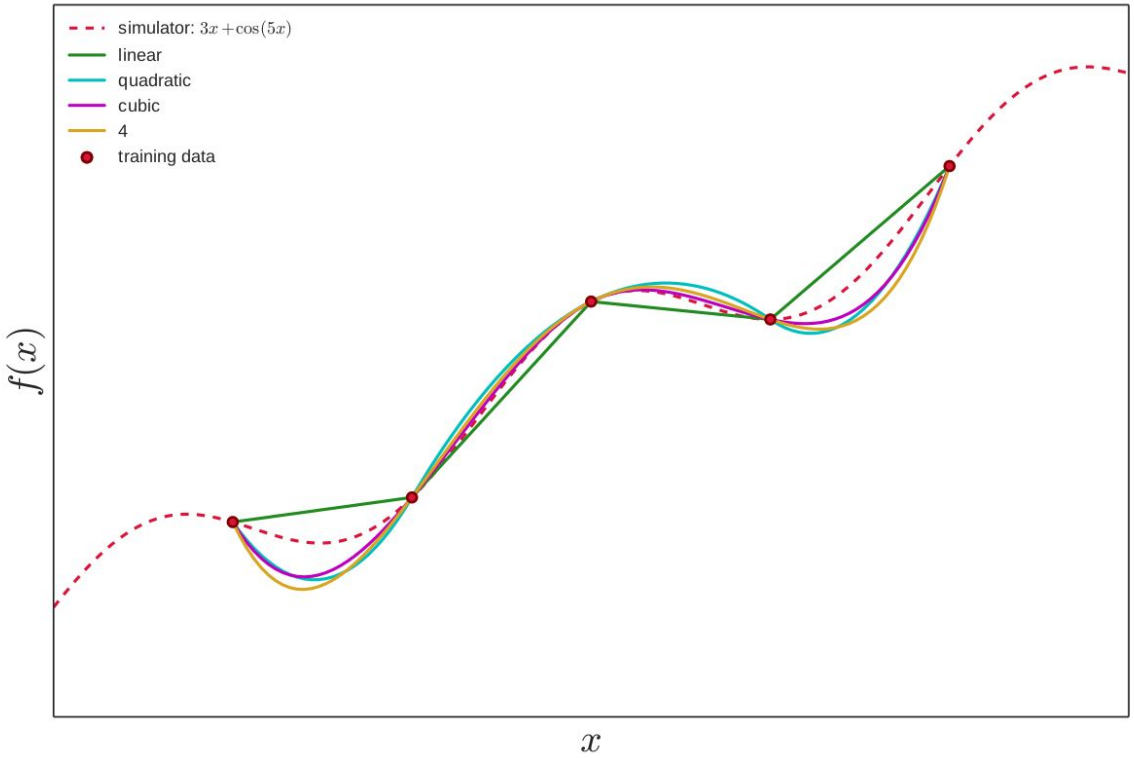
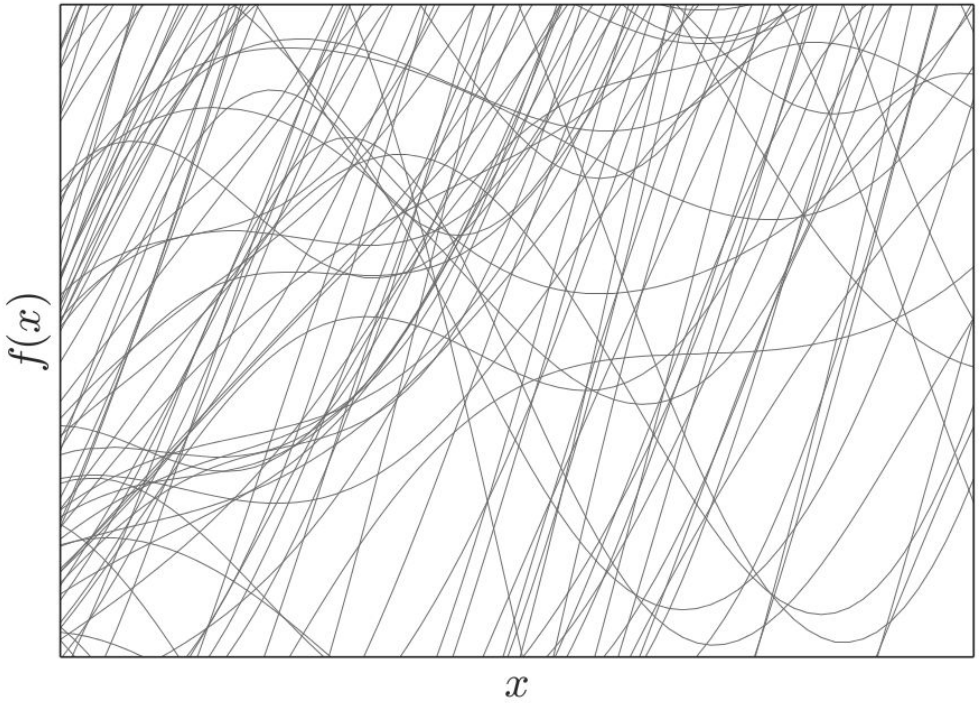
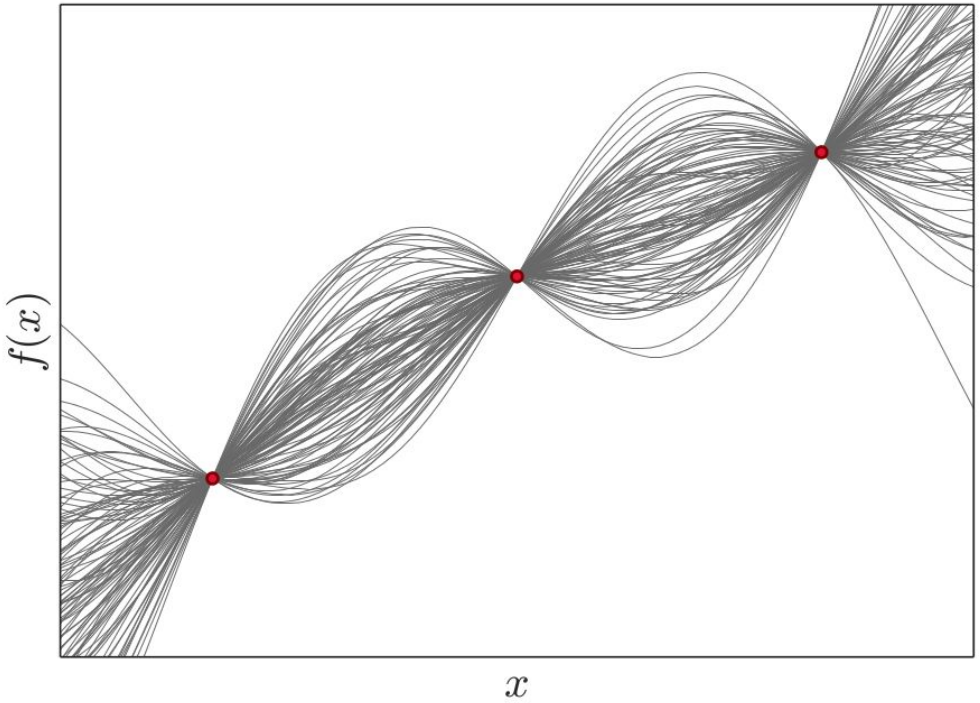
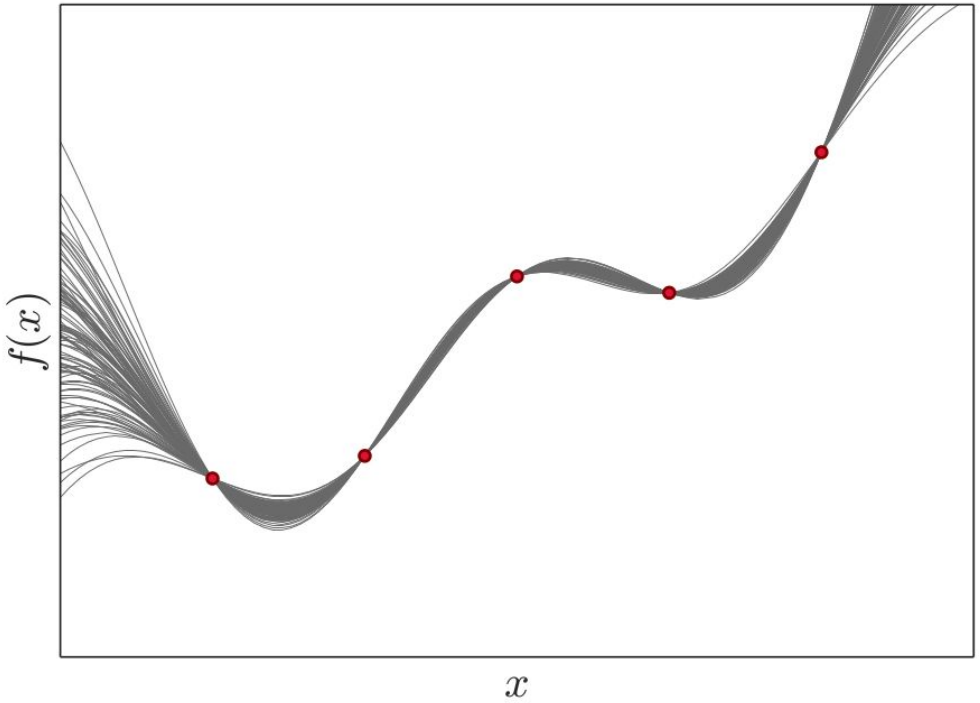
Interpolation

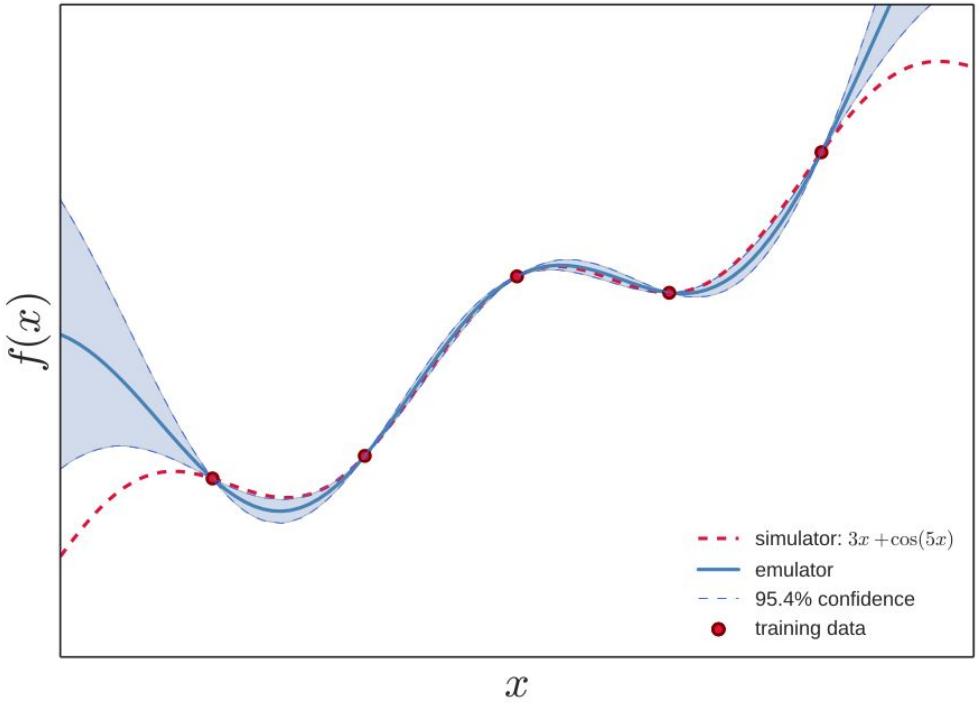



$f(x)$
x


$f(x)$
x


$f(x)$
x

Gaussian process



Gaussian process