Probabilistic sensitivity analysis

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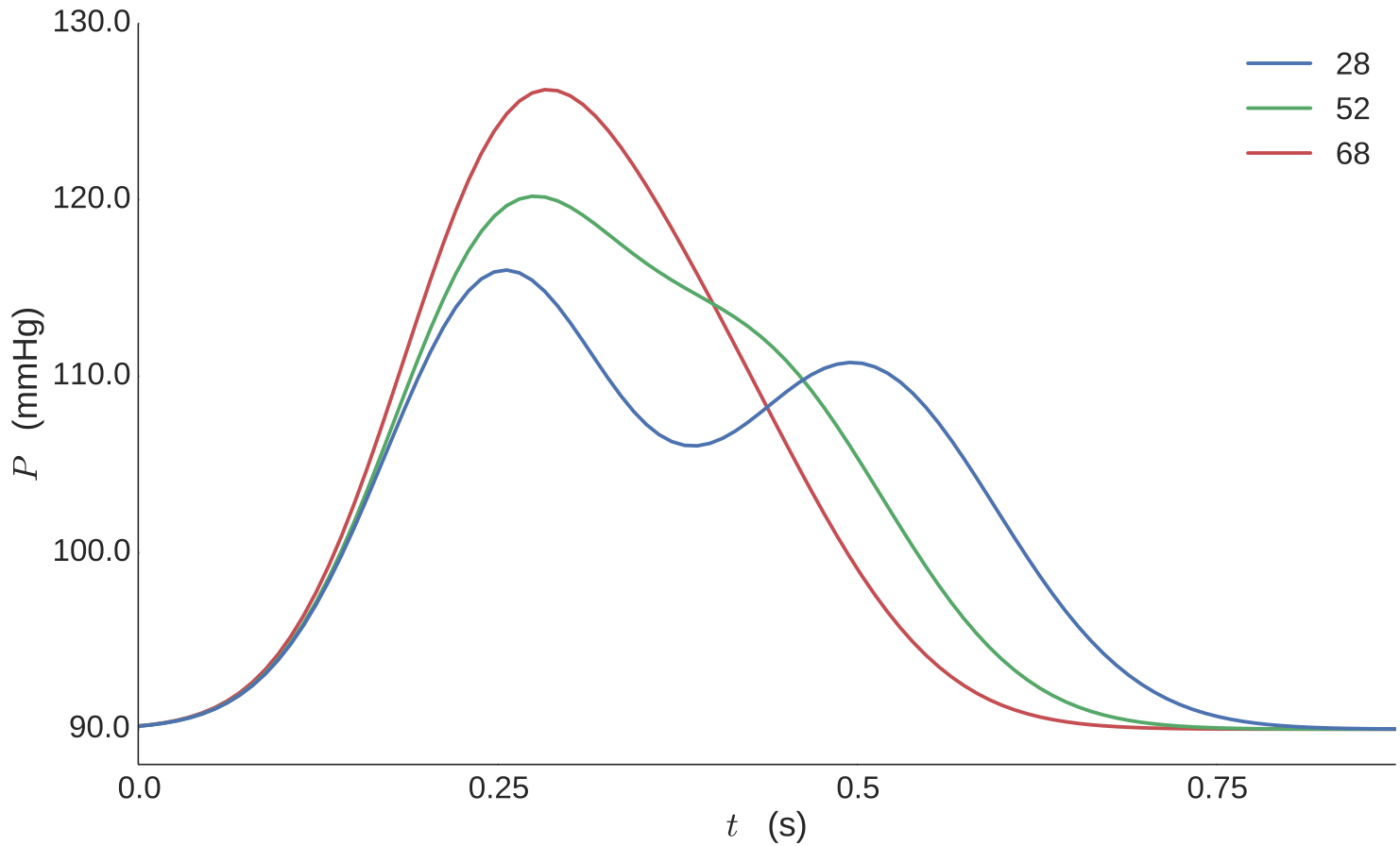
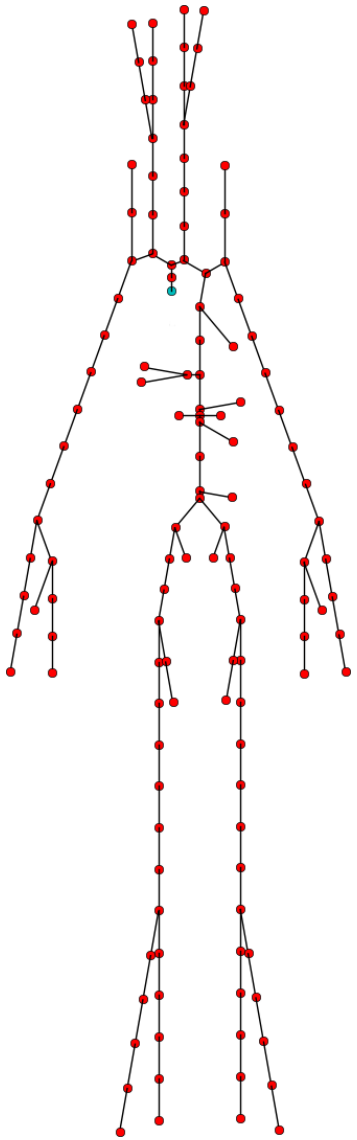
The University Of Sheffield.

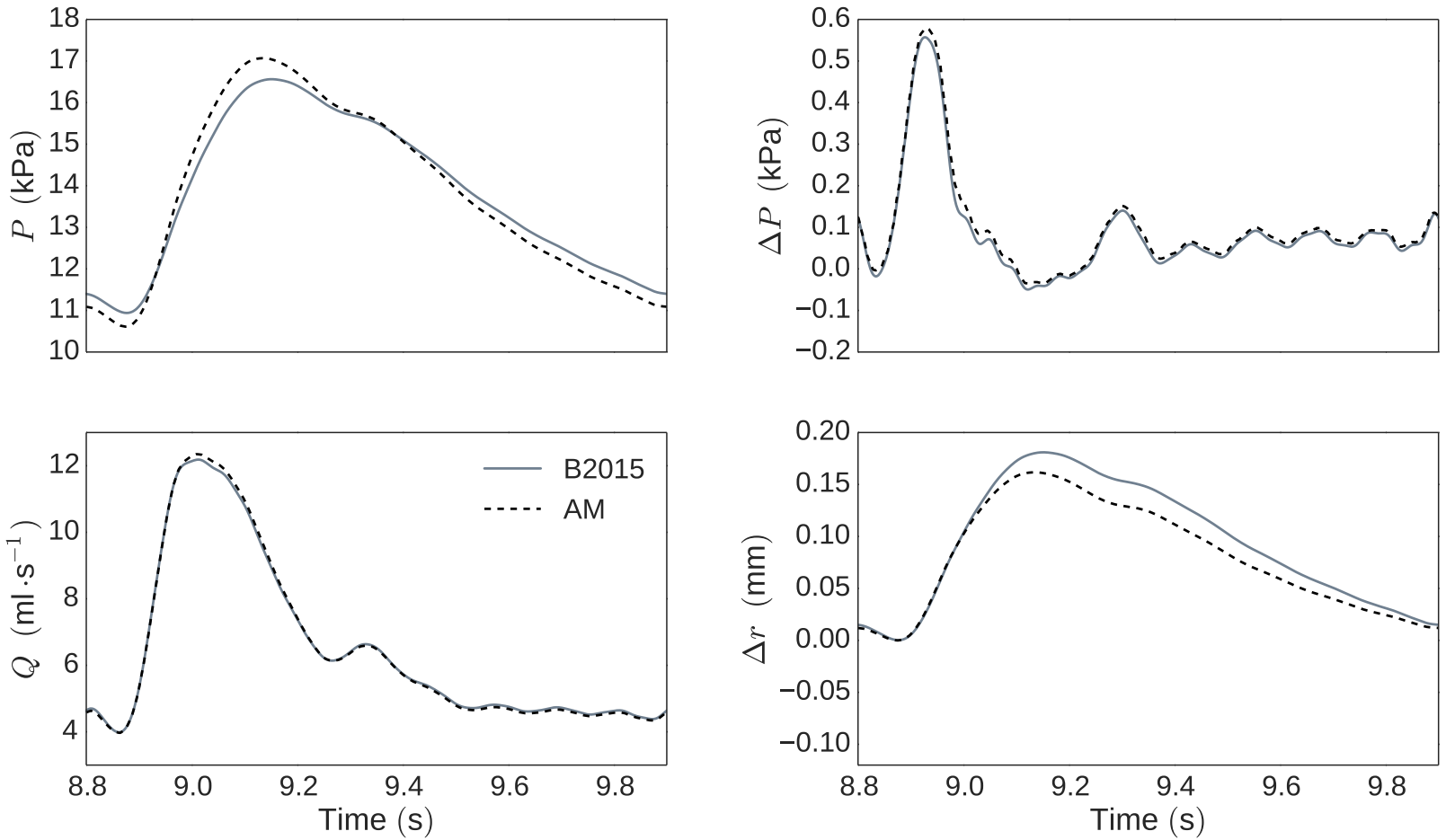
INSIGNEO Institute for *in silico* Medicine

Background

PhD aim: To model and to analyse the effects of **age** on **pulse waveforms** by means of **numerical methods**.



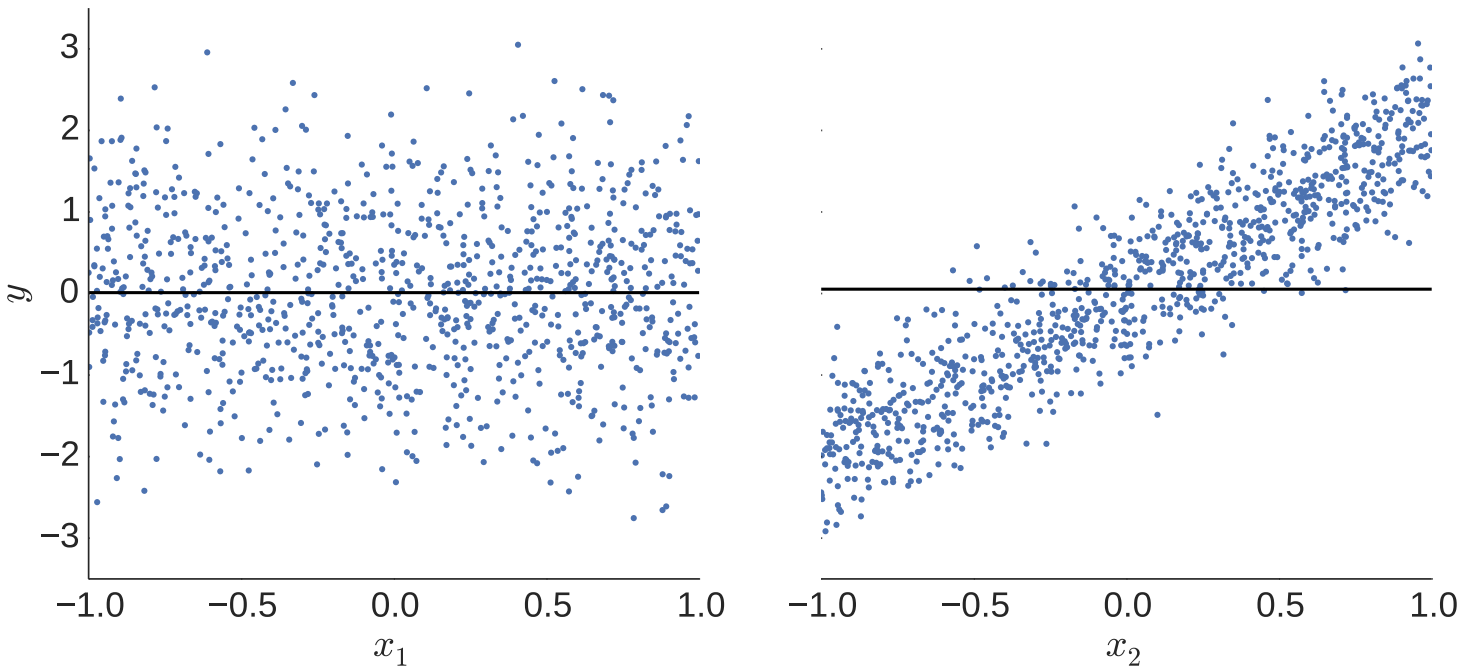




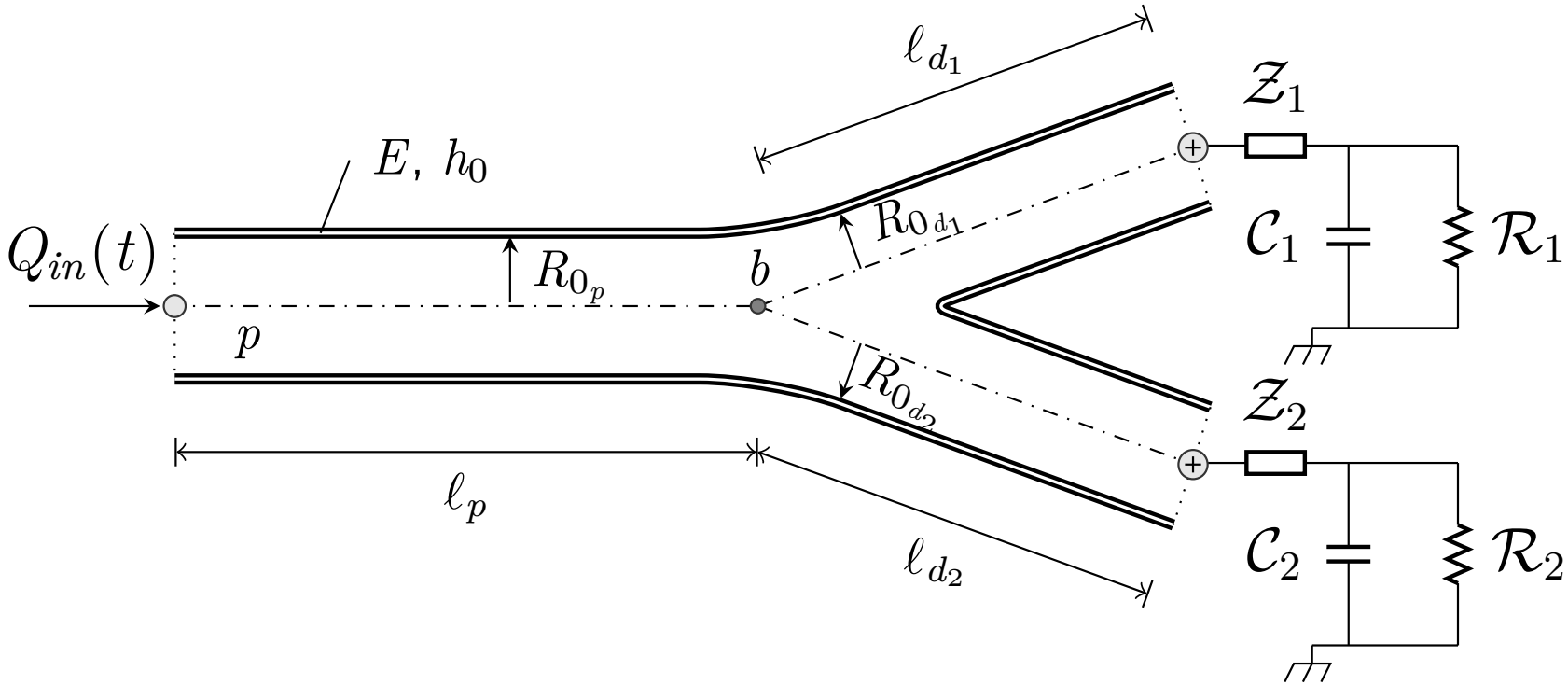
Common carotid artery benchmark. B2015: Boileau *et al.* (2015).

Sensitivity analysis

How do changes in arterial tree properties influence waveform features?



Vascular model

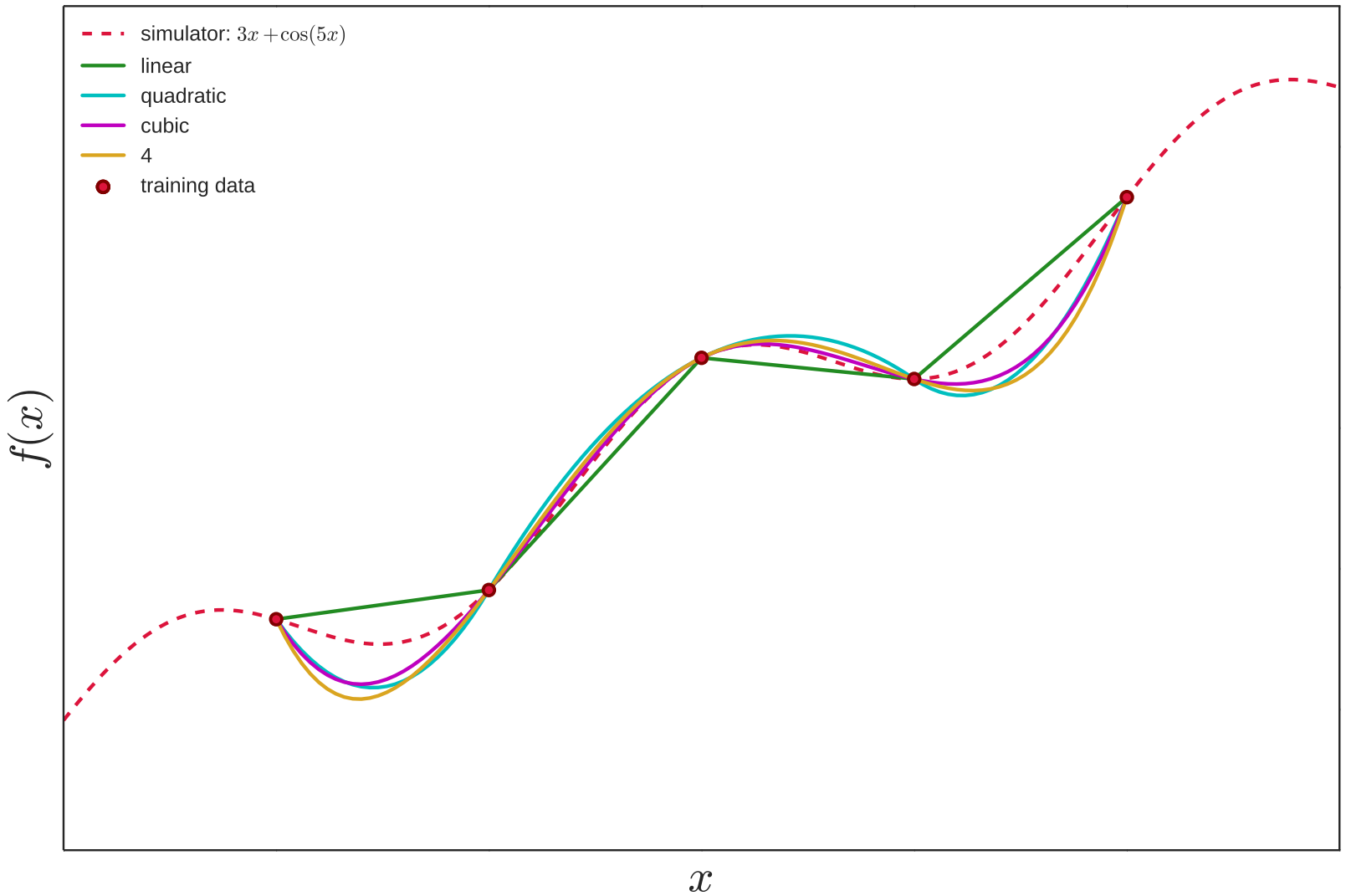


#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
input	R_{0_p}	φ	E_p	E_{d_1}	E_{d_2}	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{C}_1$	$\mathcal{C}_2$	ρ	μ	ℓ_p	ℓ_{d_1}	ℓ_{d_2}	h_p	h_{d_1}	h_{d_2}
output	min(P_p,	P_b,	P_{d_1},	P_{d_2})	max(P_p,	P_b,	P_{d_1},	P_{d_2})	$\bar{Q}_{d_1}/\bar{Q}_p$	$\bar{Q}_{d_2}/\bar{Q}_p$	PL_{p-1}	PL_{p-2}	c_p	c_{d_1}	c_{d_2}		

Feasibility

- $f(\mathbf{x}) = \mathcal{O}(d \times 10^3)$
- $d = 17$;
- $t_n \simeq 1' 15''$;
- $t_{SA} \simeq 15$ days.

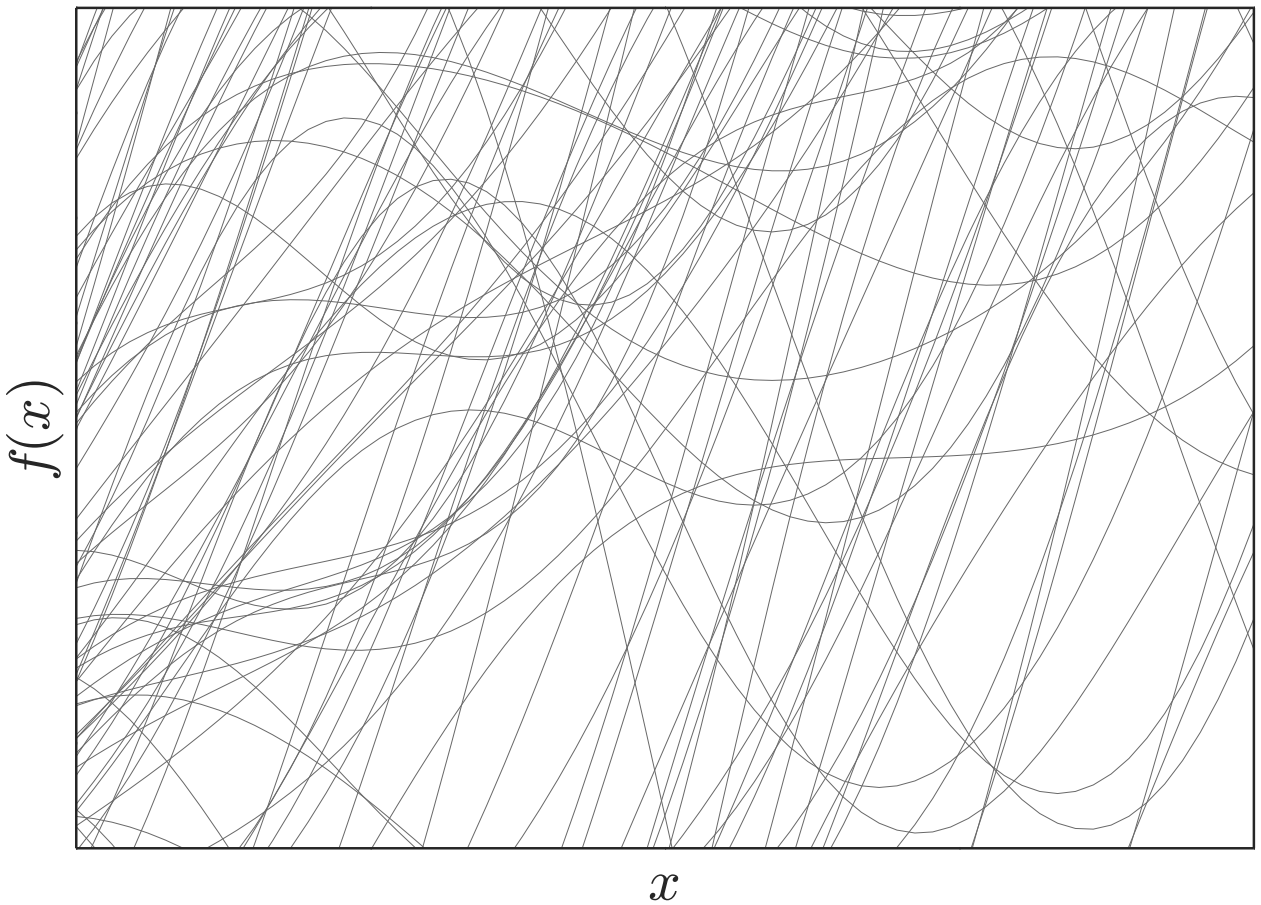
Interpolation



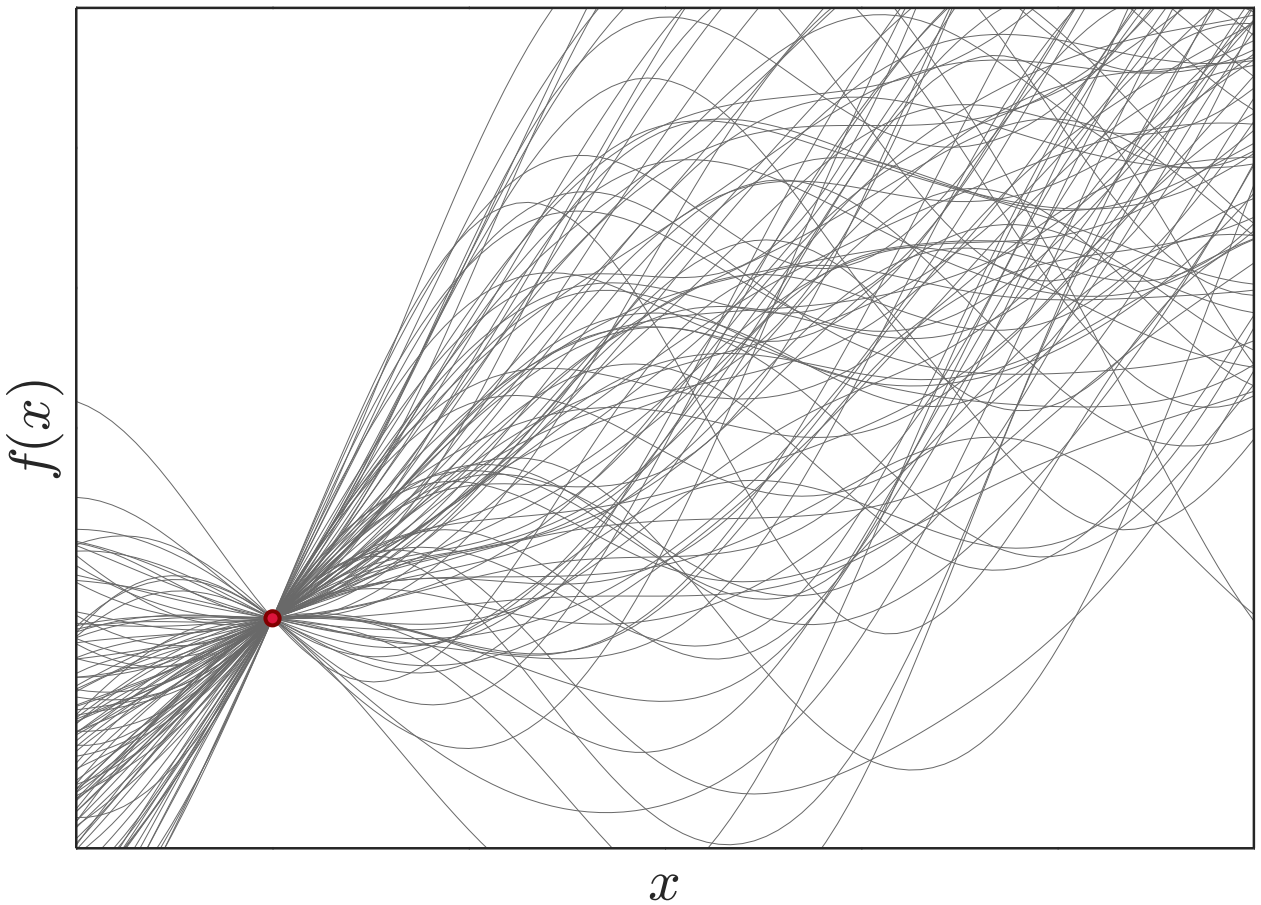
Emulator: model of a model

> *An emulator is a statistical approximation of a simulator. [...] the emulator also provides a [probability] distribution around that mean which describes how close it is likely to be to the true* $f(\mathbf{x})$.
>
> — O'Hagan (2006)

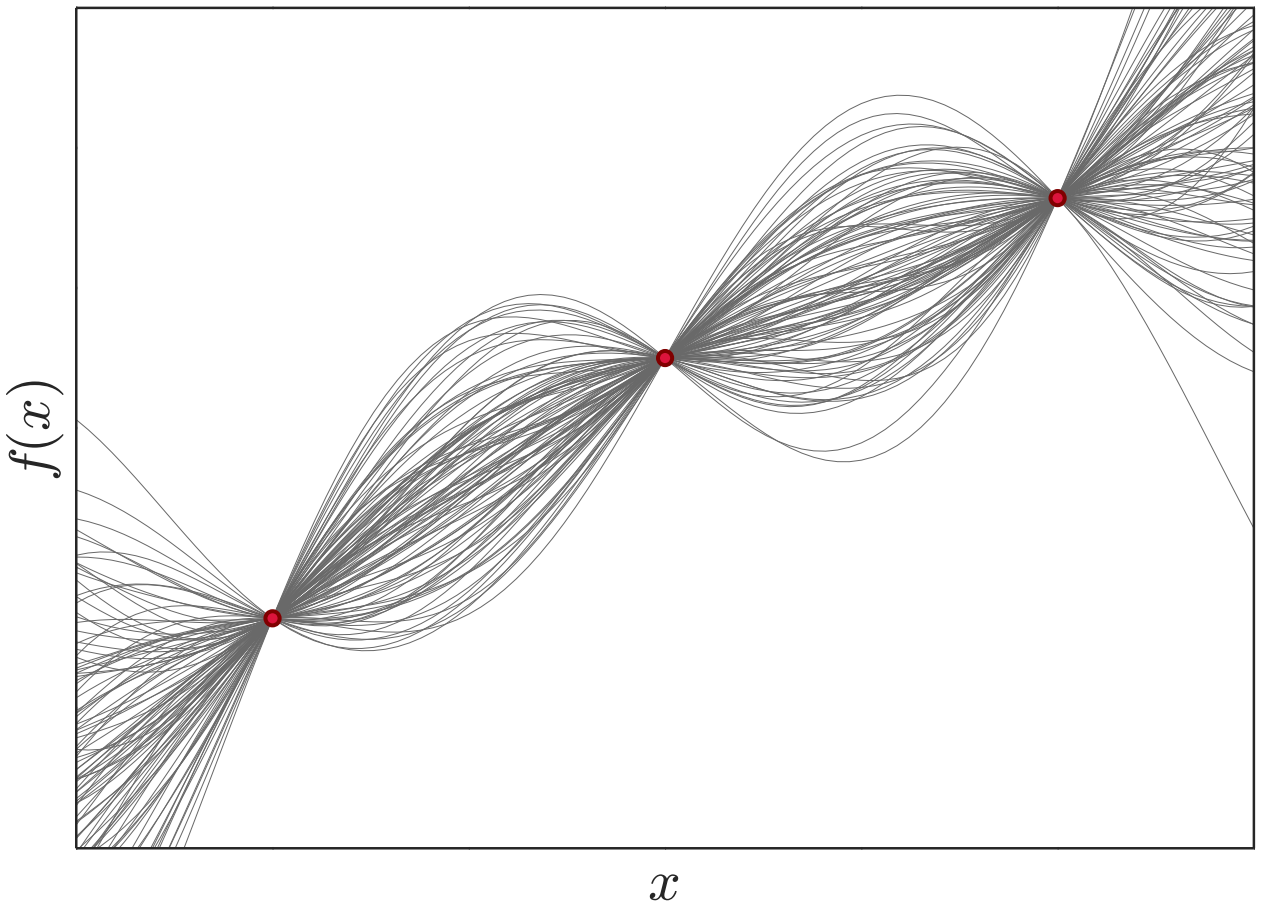
Gaussian process



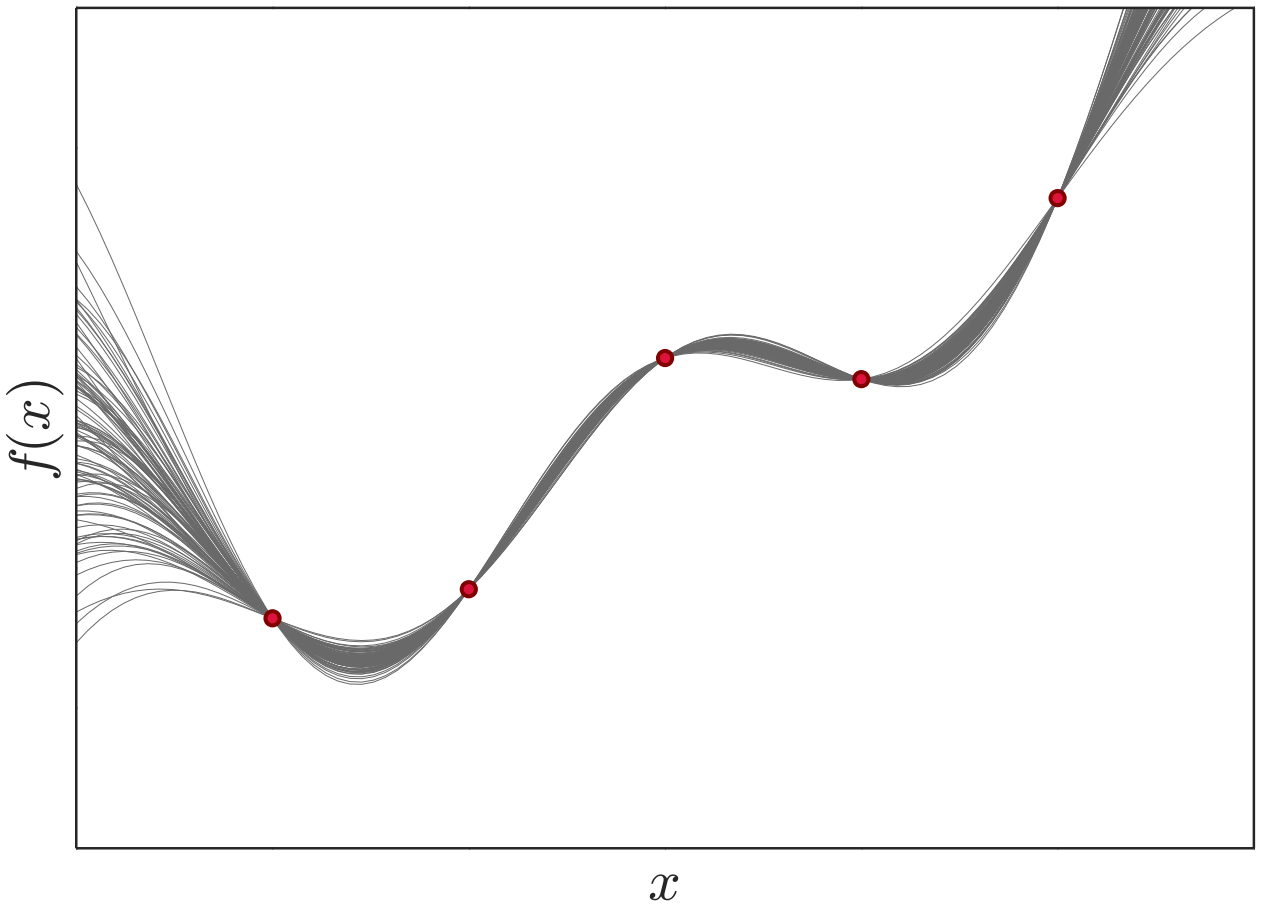
Gaussian process



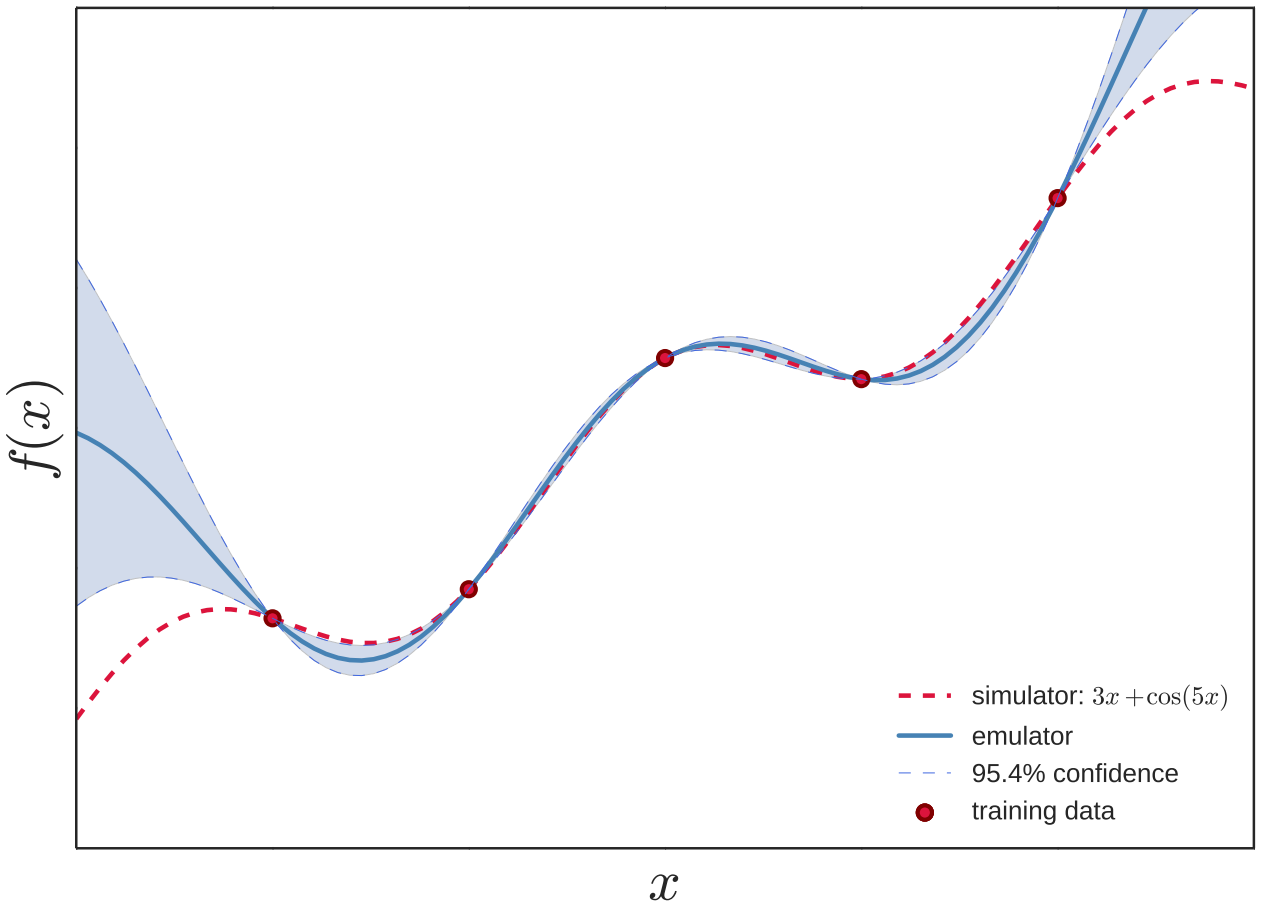
Gaussian process



Gaussian process



Gaussian process



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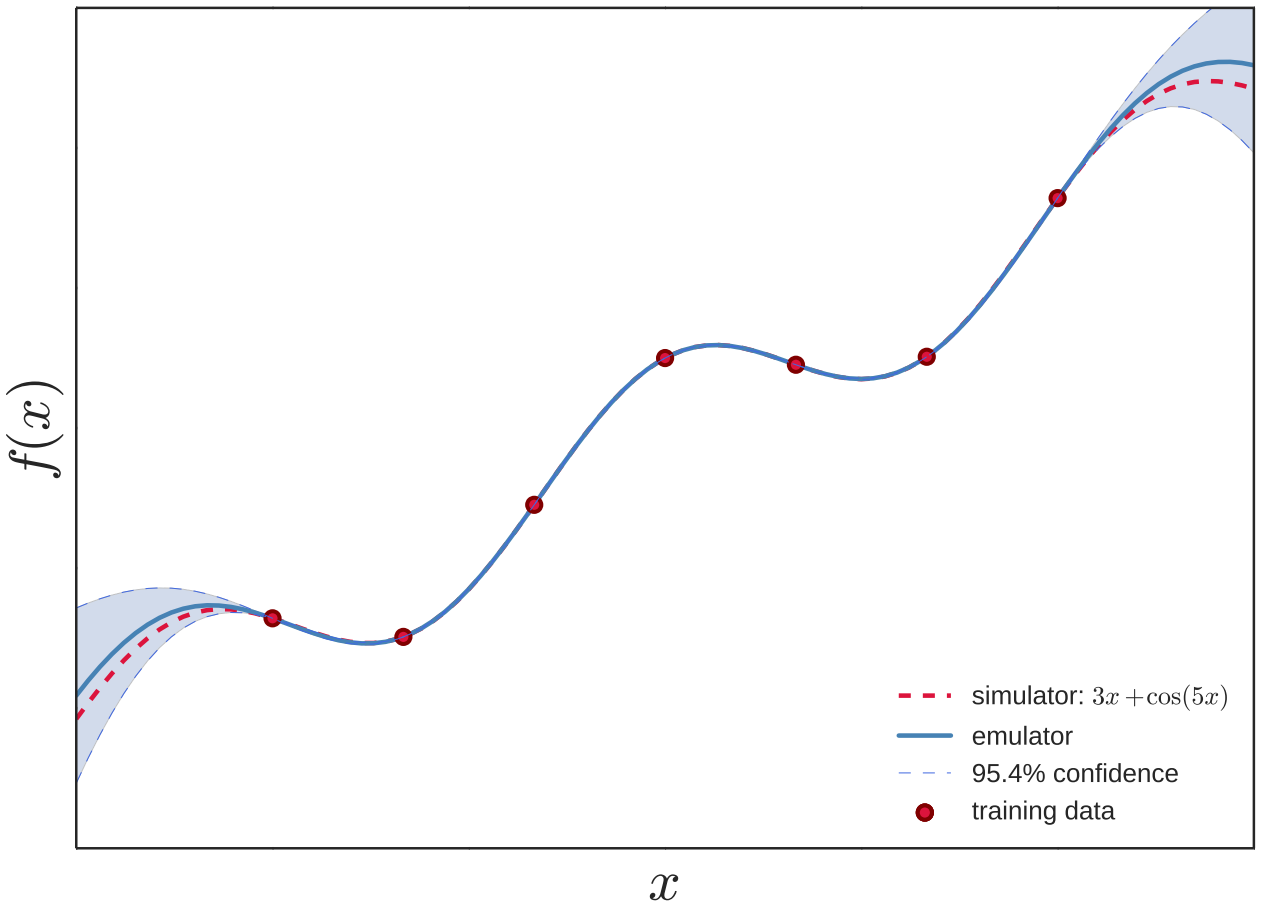

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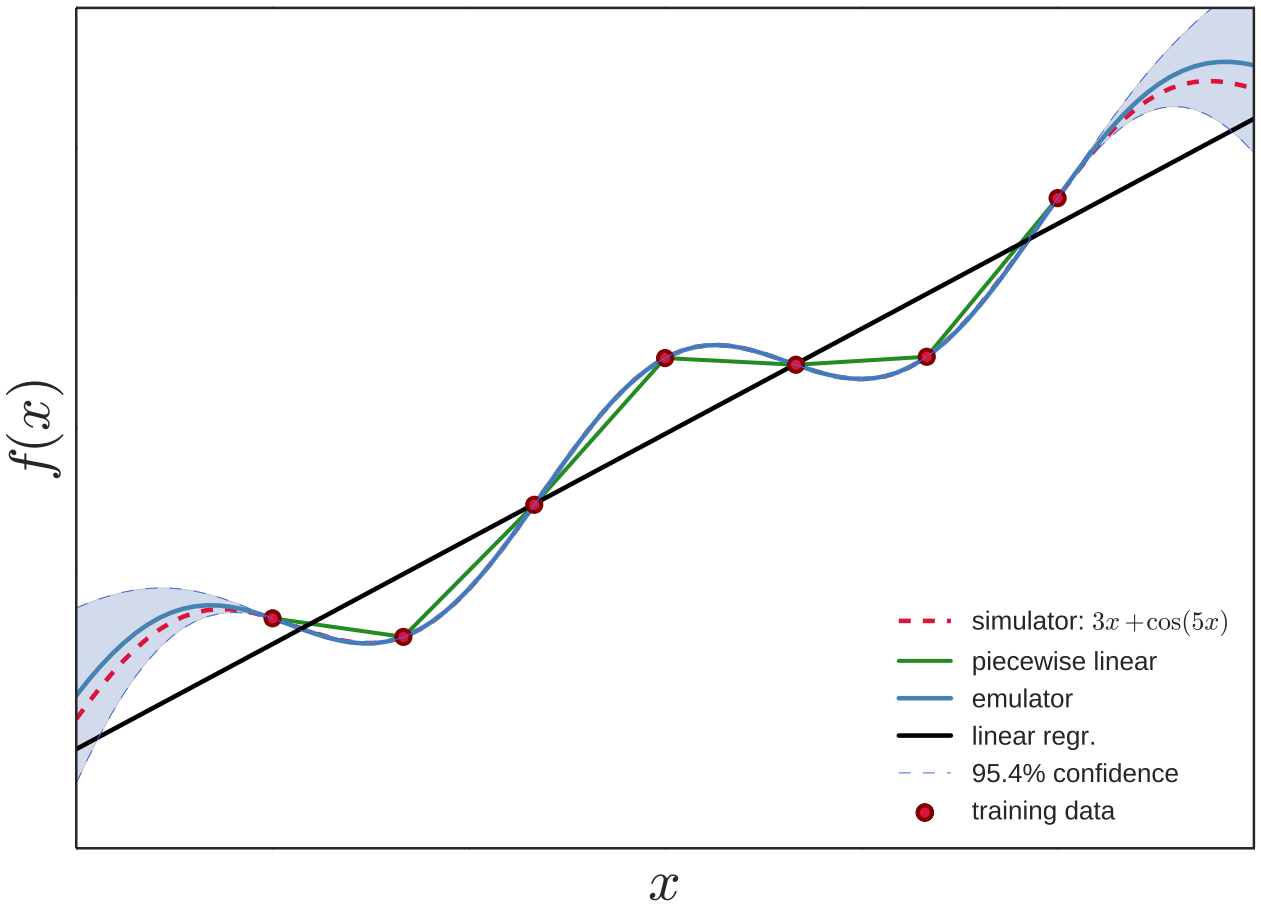
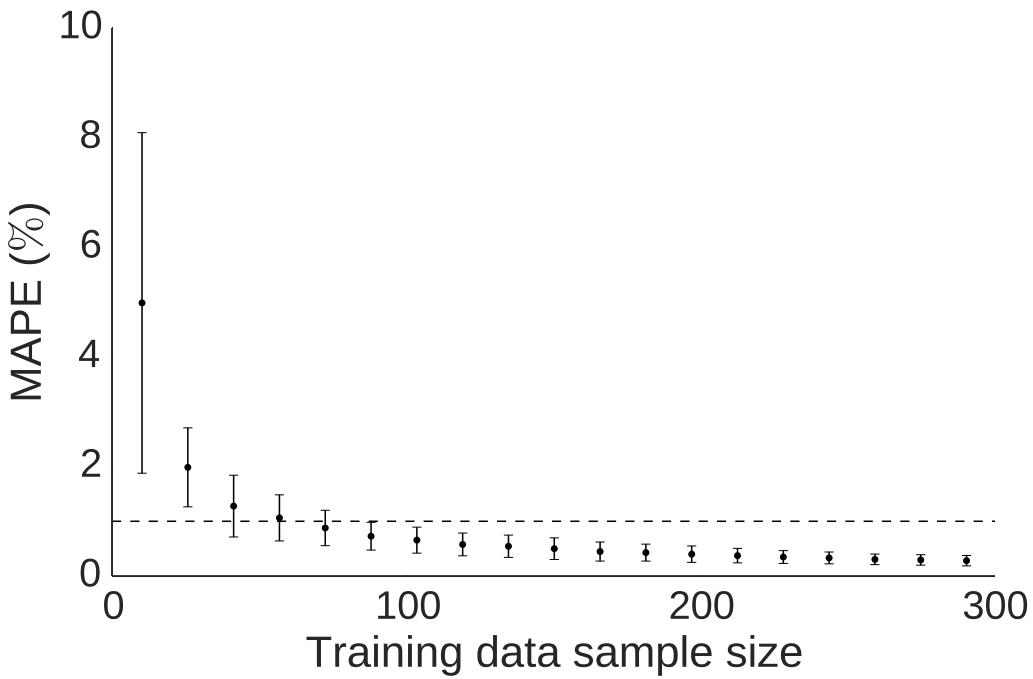


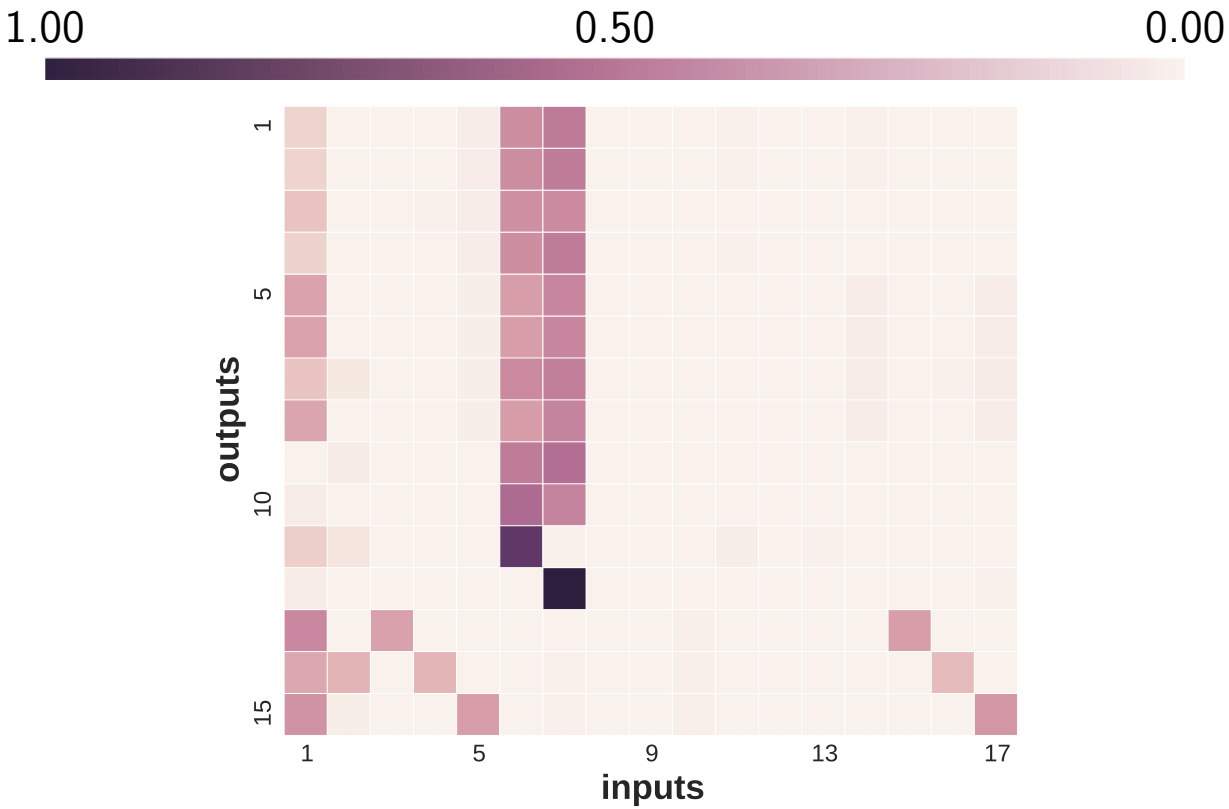
$$r = 0.95,\ p = 0.001$$

Vascular model: GP training

	$N = 70$	$N = 17000$
Numerical	1 h 25 m	14 d 17 h
GP		15.73 s



Vascular model: sensitivity indices



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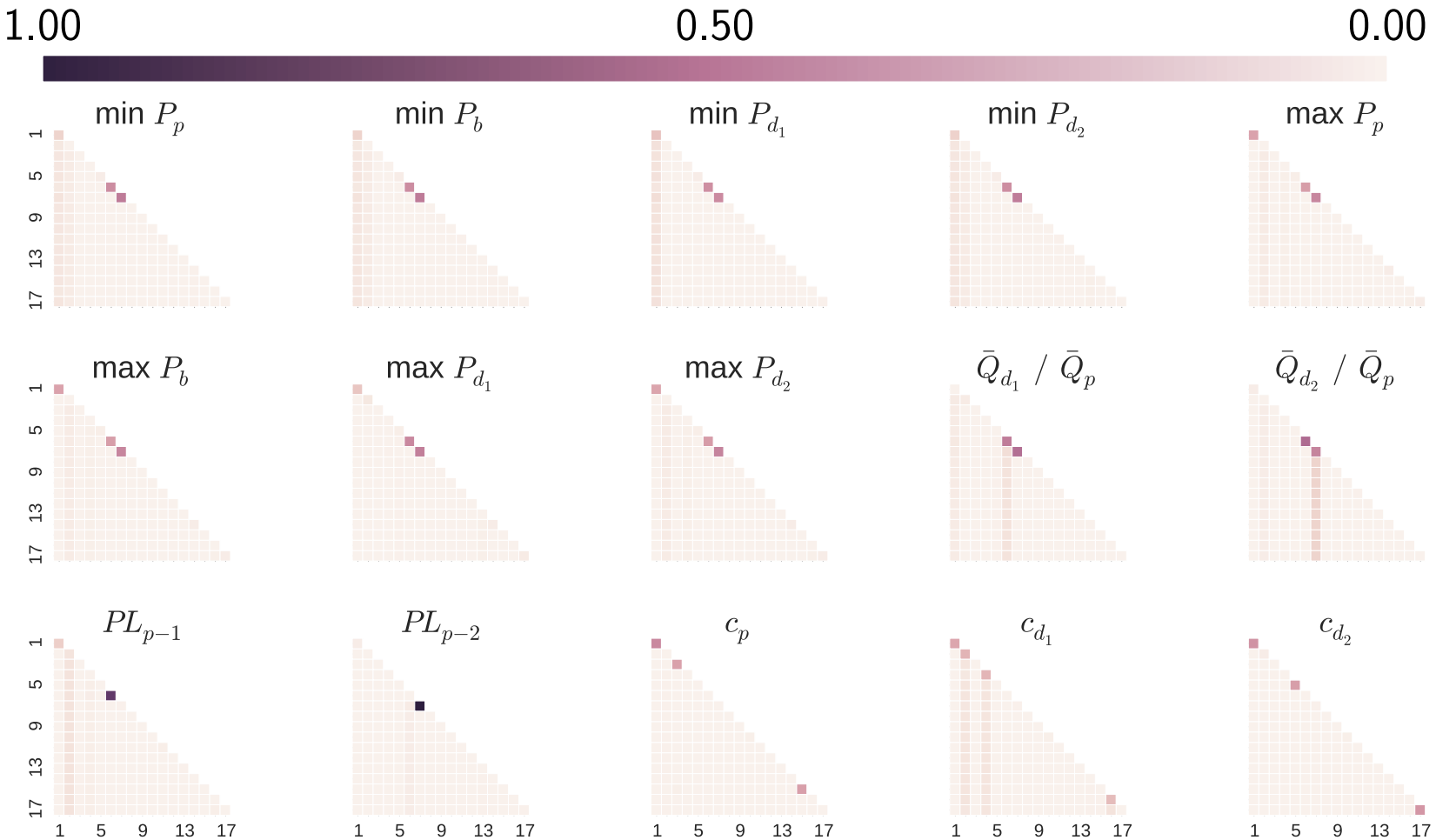
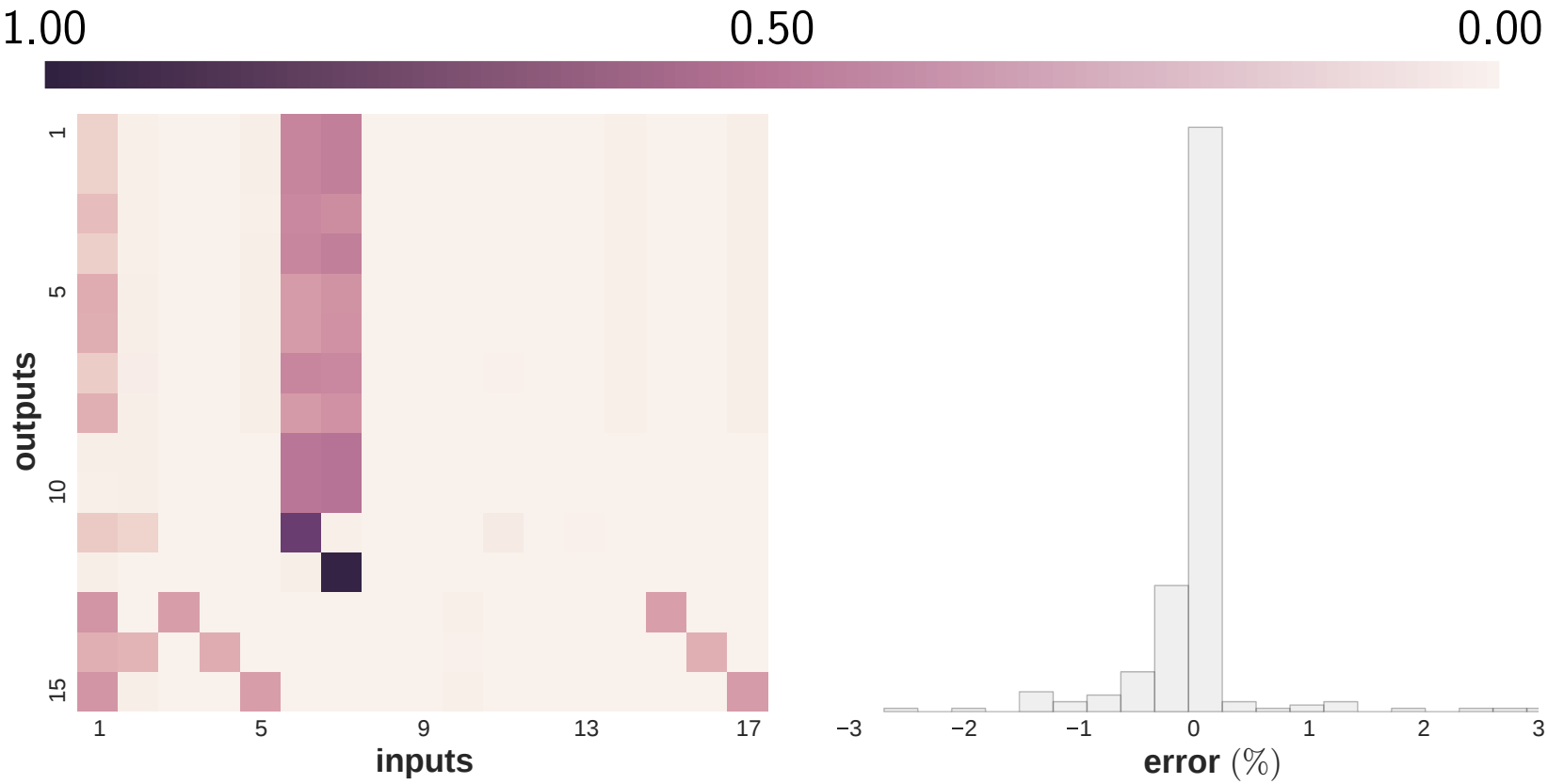
output	$\min(P_p,$	$P_b,$	$P_{d_1},$	$P_{d_2})$	$\max(P_p,$	$P_b,$	$P_{d_1},$	$P_{d_2})$	$\bar{Q}_{d_1}/\bar{Q}_p$	$\bar{Q}_{d_2}/\bar{Q}_p$	PL_{p-1}	PL_{p-2}	c_p	c_{d_1}	c_{d_2}		

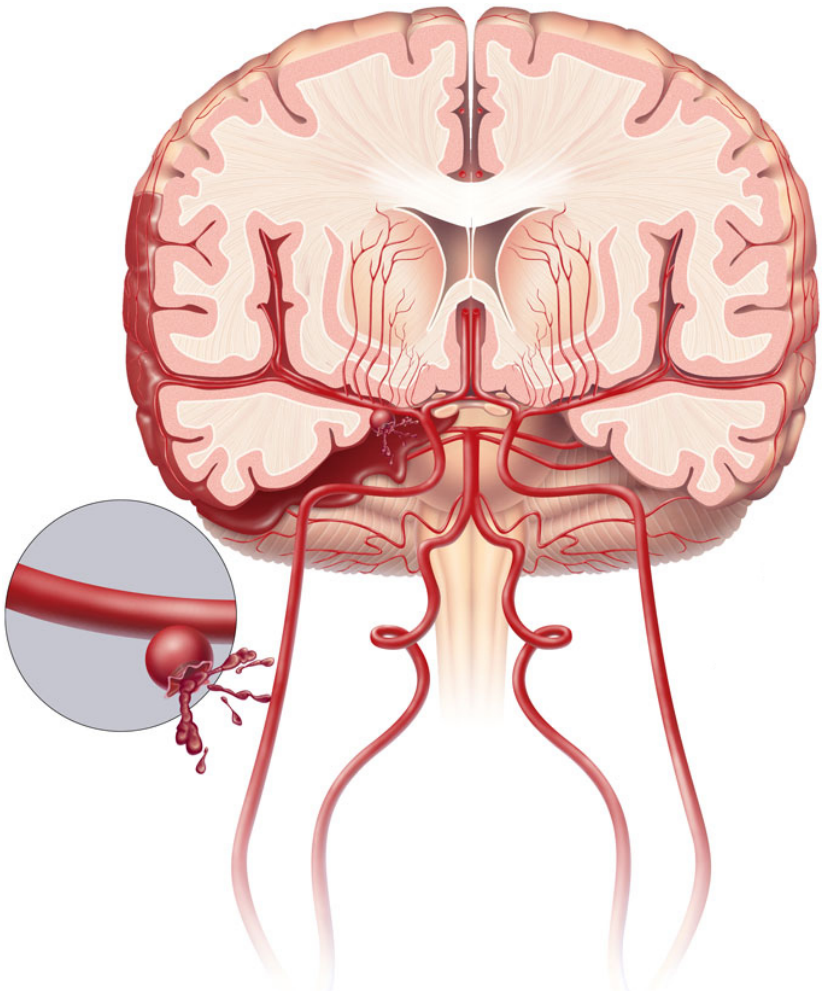
Vascular model: inputs collaboration



Vascular model sensitivity analysis: MC validation



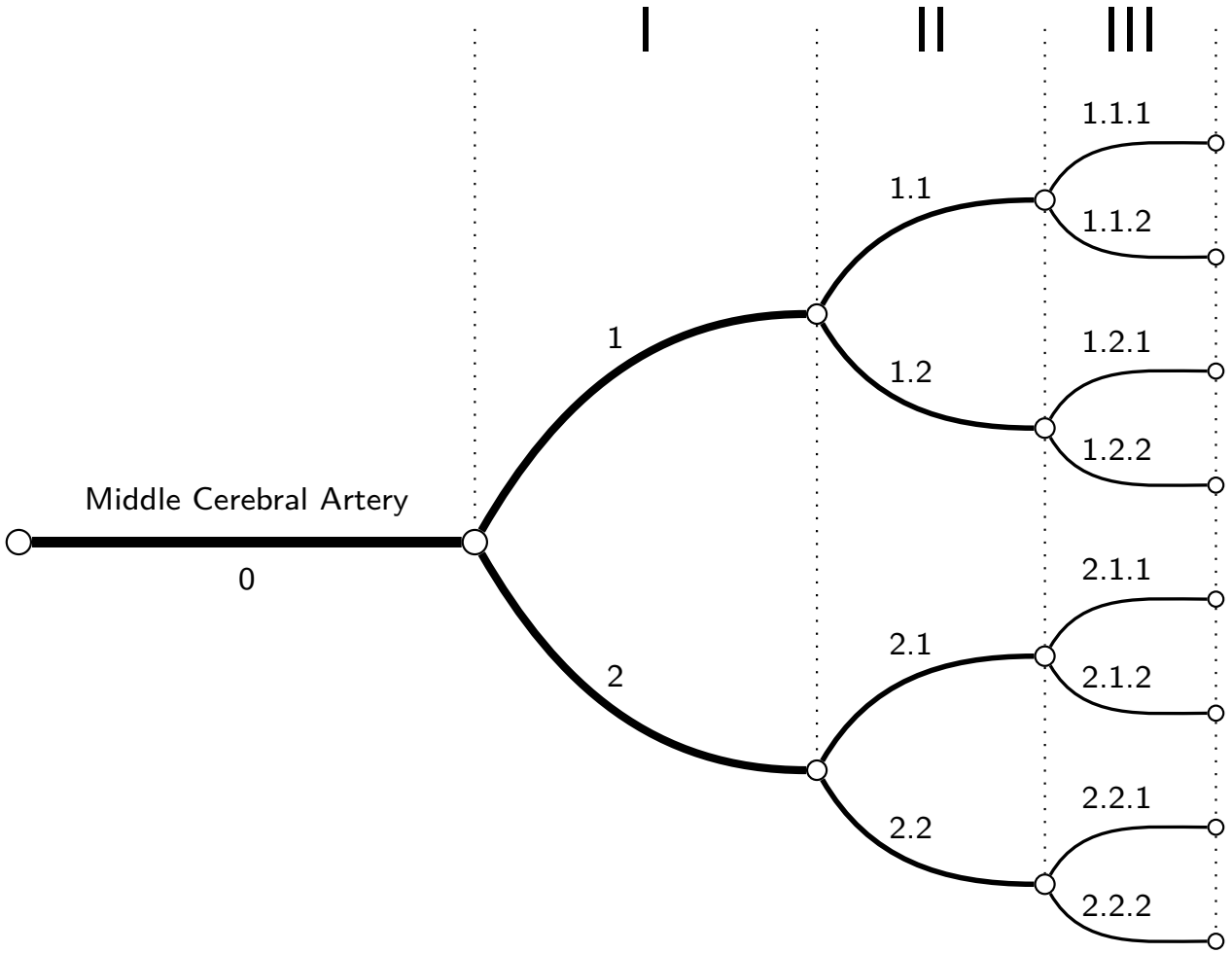
Future work: vasospasm



Subarachnoid hæmorrage.
The Internet Stroke Center (2016)

- Arterial narrowing;
- aneurysm rupture in subarachnoid space;
- peaks in severity 1 week after subarachnoid hæmorrage;
- possible cerebral ischemia;
- transcranial doppler: blood velocity measurement.

Future work: methodology



Thank you!

thoughts?

- Boileau, E., *et al.* A benchmark study of numerical schemes for one dimensional arterial blood flow modelling. *International journal for numerical methods in biomedical engineering* 2015.
- O'Hagan A. Bayesian analysis of computer code outputs: a tutorial. *Reliability Engineering & System Safety* 2006; **91**(10):1290-1300.
- Findlay JM, Nisar J, Darsaut T. Cerebral vasospasm: a review. *Canadian Journal of Neurological Sciences* 2015; **2**:1-8.