Vascular networks sensitivity analysis

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CURE meeting
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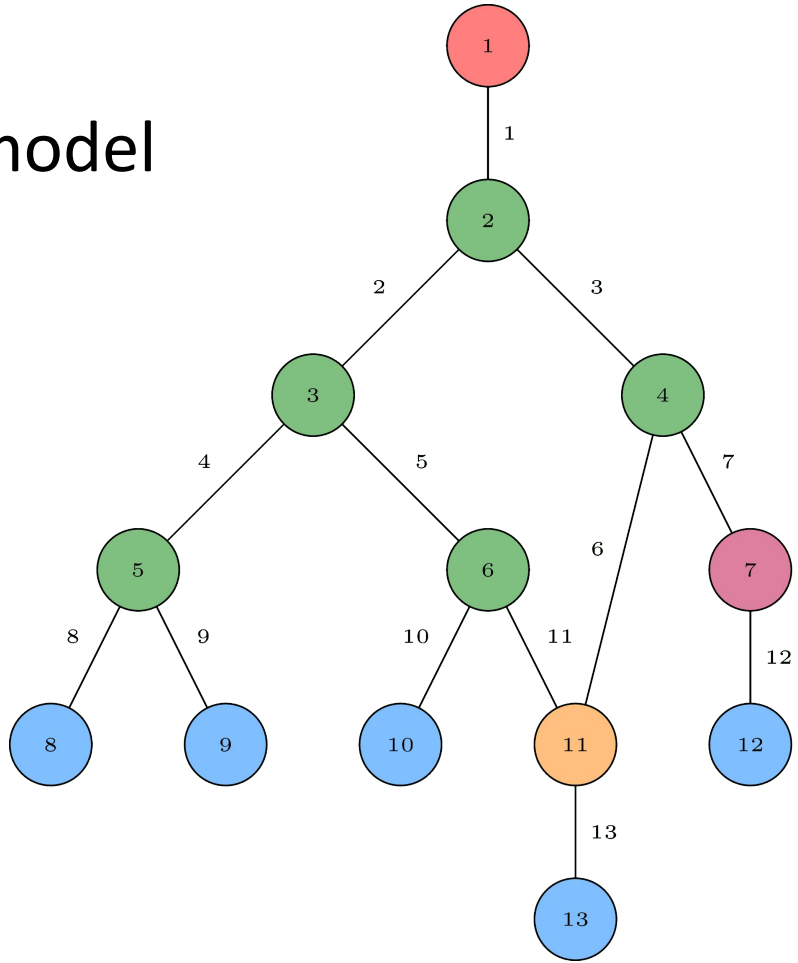
Outline

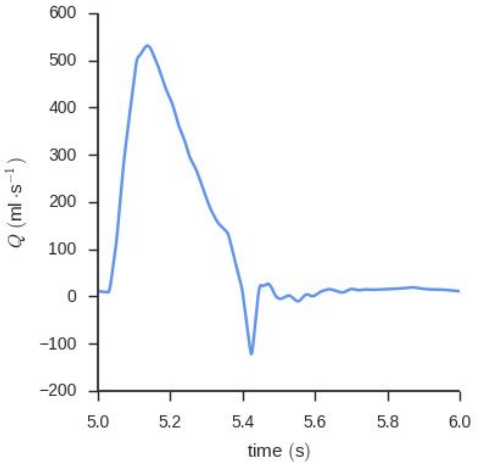
- 1D vascular model
- Pulse waveform analysis
- Sensitivity analysis
- The curse of dimensionality
- Gaussian process
- Application: cerebral vasospasm

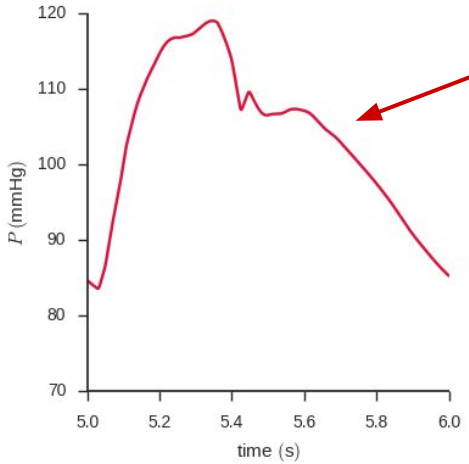
1D model

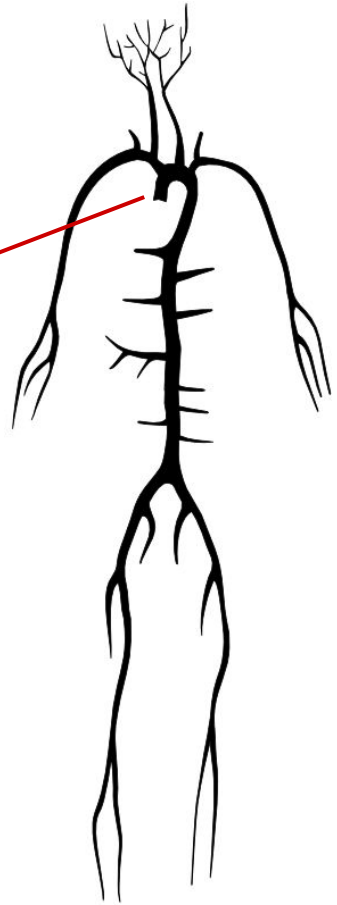


$$\begin{cases} \dfrac{\partial A}{\partial t} + \dfrac{\partial (Au)}{\partial x} = 0, \\ \dfrac{\partial (Au)}{\partial t} + \dfrac{\partial \left(Au^2\right)}{\partial x} + \dfrac{A}{\rho}\dfrac{\partial P}{\partial x} = -8\pi\dfrac{\mu}{\rho}u, \\ P = P_{ext} + \beta\left[\left(\dfrac{A}{A_0}\right)^{1/2} - 1\right], \quad \beta = \sqrt{\dfrac{\pi}{A_0}}\dfrac{Eh_0}{1-\sigma^2} \end{cases}$$

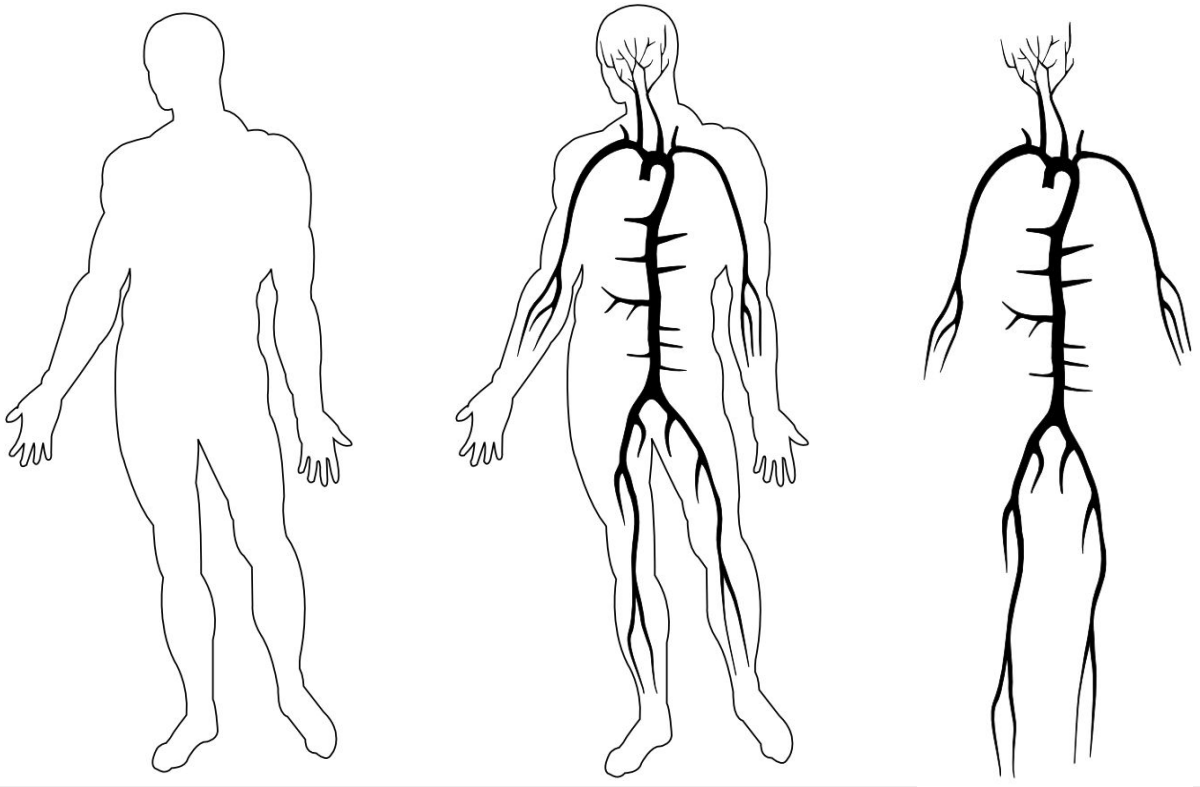
Pulse waveform analysis



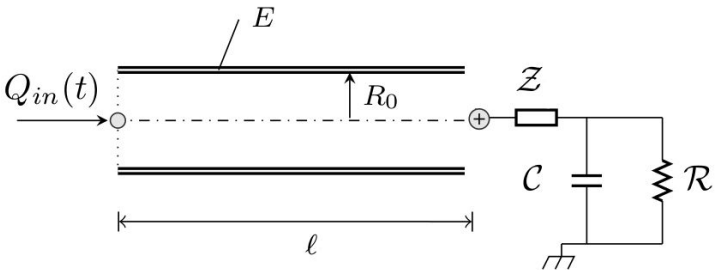




Patient specific model



However…

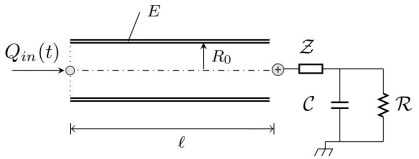


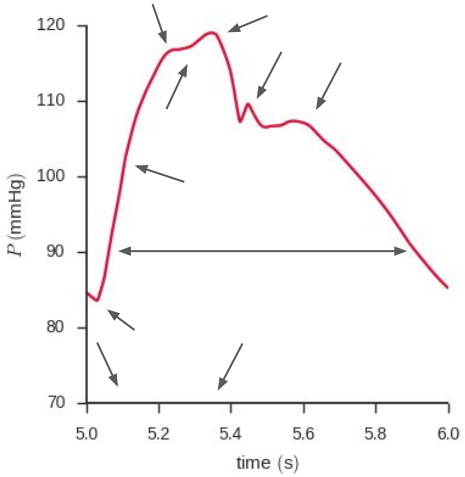
Single vessel + capillary bed model

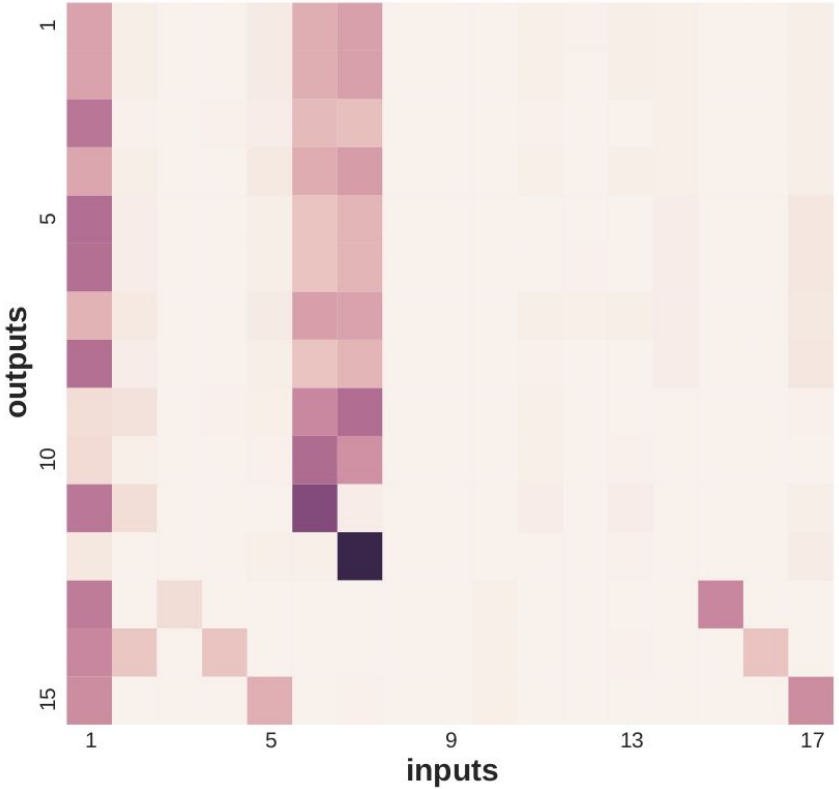
Reymond (2009)

Arterial Segment Name	Arterial Segment Number (Right/Left)	Arterial Segment Length, mm	Proximal Lumen Diameter, mm	Distal Lumen Diameter, mm	Distensibility, 10^{-3} l/mmHg	Viscelasticity coeff. (ā)	Terminal Resistance ($R_1 + R_2$), mmHg·s·ml^{-1}	Terminal Compliance (C_T), 10-5 ml/mmHg
Ascending aorta 1	1	5	29.4	29.3	5.46	0.05		
Aortic arch A	2	20	25.1	24.0	4.90	0.05		
Brachiocephalic	3	34	20.2	18.0	4.22	0.05		
Subclavian A	4/19	34	11.5/11.0	9.0/8.5	2.90/2.81	0.09/0.10		
Common carotid	5/15	94/139[b]	13.5/12.0	7.0/6.0[e]	2.93/2.68	0.09/0.10		
Vertebral	6/20	149/148	3.7[d,i,j]	2.8[c]	1.46	0.14		
Subclavian B, axillary, brachial	7/21	422	8.1	4.7	2.19	0.12		
Radial	8/22	235	3.7/3.5	3.1/2.8	1.49/1.43	0.14	39.7	702.9
Ulnar A	9/23	67	3.7/4.3	3.4/4.3	1.53/1.72	0.14/0.13		
Interosseous	10/24	79	2.1/1.8	1.8	1.08/1.03	0.14/0.15	633.8	44.0
Ulnar B	11/25	171	3.2/4.1	2.8/3.7	1.39/1.62	0.14/0.13	39.7	702.9
Internal carotid	12/16	178	5.7/5.3[d,i,j]	4.3/4.1[c,j]	1.89/1.82	0.13		
External carotid 1	13/17	41[b]	5.0/4.7	4.5/4.3	1.83/1.77	0.13		
Aortic arch B	14	39	21.4	20.8	4.48	0.05		
Thoracic aorta A	18	52	20.0	18.9	4.26	0.05		
Intercostals	26	80	12.6	9.5	3.04	0.09	10.5	2670.1
Thoracic aorta B	27	104	16.5	12.9	3.60	0.07		
Abdominal aorta A	28	53	12.2	12.2	3.22	0.08		
Celiac A	29	20	7.8	6.9	2.38	0.11		
Celiac B	30	20	5.2	4.9	1.90	0.13		
Hepatic	31	66	5.4	4.4	1.87	0.13	27.3	1022.5
Gastric	32	71	3.2	3.0	1.42	0.14	40.7	686.1
Splenic	33	63	4.2	3.9	1.66	0.13	17.4	1599.8
Superior mesenteric	34	59	7.9	7.1	2.41	0.11	7.0	3991.0
Abdominal aorta B	35	20	11.5	11.3	3.09	0.09		
Renal	36/38	32	5.2	5.2	1.93	0.12	8.5	3284.6
Abdominal aorta C	37	20	11.8	11.8	3.16	0.08		
Abdominal aorta D	39	106	11.6	11.0	3.07	0.09		
Inferior mesenteric	40	50	4.7	3.2	1.64	0.13	51.7	539.5
Abdominal aorta E	41	20	10.8	10.4	2.96	0.09		
Common iliac	42/43	59	7.9	7.0	2.39	0.11		
External iliac	44/50	144	6.4	6.1	2.15	0.12		
Inner iliac	45/51	50	4.0	4.0	1.65	0.13	59.7	467.7
Femoral	46/52	443	5.2	3.8	1.77	0.13		
Deep femoral	47/53	126	4.0	3.7	1.61	0.13	35.9	778.1
Posterior tibial	48/54	321	3.1	2.8	1.38	0.14	35.9	778.1
Anterior tibial	49/55	343	2.6	2.3	1.24	0.14	42.0	664.0
Basilar artery 2	56	20	4.0[d,h]	3.6	1.60	0.31		
Superior cerebellar	57/58	10	1.7[f]	1.4	0.93	0.33	200.8	3.6
Basilar artery 1	59	5	3.1[g]	2.7	1.36	0.32		
Post. cerebral 1	60/61	2	1.9[c,d,f,j]	1.9	1.05	0.33		
Post. communicating	62/63	4	1.2[c,f,j]	1.2	0.78	0.33		
Post. cerebral 2	64/65	59	2.0[g]	1.8	1.12	0.32	80.5	5.8
ICA distal PCo–ant. chor. seg.	66/67	2	3.9	3.8	1.62	0.31		
Ant. cerebral 1	68/69	12	2.1[c,d,f,h,j]	2.0	1.10	0.32		
Middle cerebral M1	70/73	8	3.0[c,f,j]	2.8	1.36	0.32		
MCA M2 sup. fr. cist. sylvian bif.	71/74	71	2.0	1.0	0.92	0.33	75.2	2.8
MCA M2 inf. br. dist. sylvian bif.	72/75	70	2.0	1.0	0.92	0.33	75.2	2.8
Ant. cerebral A2	76/78	24	1.8	1.7	0.99	0.33	80.5	4.7
Ant. communicating	77	2	1.3[f,j]	1.3	0.84	0.33		
Int. car. sinus	79/81	11	4.3	3.9	1.67	0.31		
Ophthalmic	80/82	11	1.0[g]	0.5	0.60	0.33	200.8	0.4
External carotid 2	83/85	61	4.0	3.5	1.59	0.13		
Sup. thy. asc. ph. lyng. fac. occ.	84/86	101	2.0	1.0	0.92	0.15	225.6	5.9
Superficial temporal	87/89	61	3.2	3.0	1.42	0.14		
Maxillary	88/90	91	2.2	1.0	0.95	0.15	188.0	5.0
Superficial temporal frontal br.	91/93	100	2.2	1.4	1.02	0.15	188.0	8.2
Superficial temporal parietal br.	92/94	101	2.2	1.4	1.02	0.15	188.0	7.6
Ascending aorta 2	95	35	29.3	28.8	5.42	0.05		
Right coronary RCA	96	53.7[g]	3.6[a]	2.6[a]	1.42	0.14	55.6	26.6
Left main coronary LCA	97	5[a]	4.9	4.7	1.84	0.13		
Left anterior descending cororanry LAD	98	47[g]	3.8[a]	1.5[a]	1.29	0.14	45.1	26.6
Left circumflex LCx	99	26[g]	3.5[a]	3.1[a]	1.47	0.14	45.1	26.6
Ant. choroidal	100/102	36	1.5	1.3	0.88	0.15	150.4	15.4
ICA distal ent. chor.–M1 seg.	101/103	2	3.8	3.8	1.61	0.13		

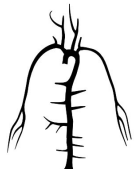
Sensitivity analysis

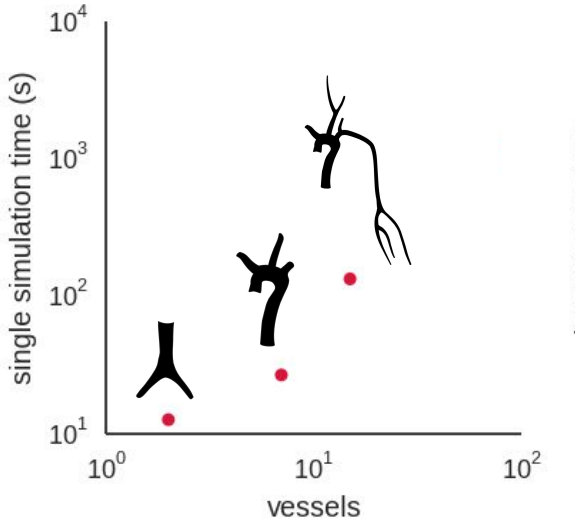






The *curse of dimensionality*



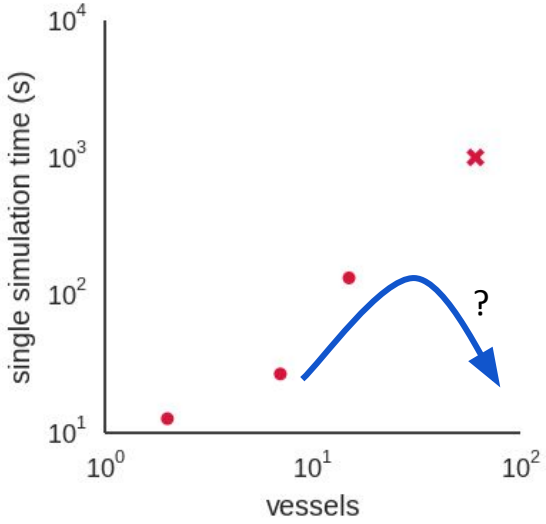


IS

Number of simulations = number of input parameters X 1000

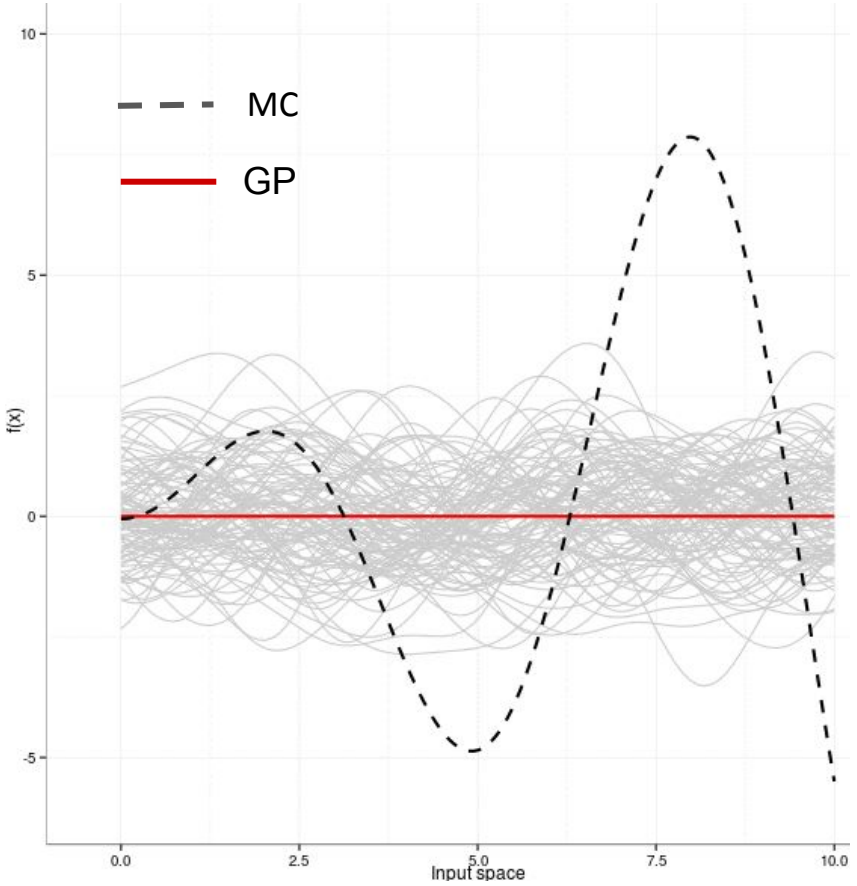
Saltelli (2001)

Faster code?

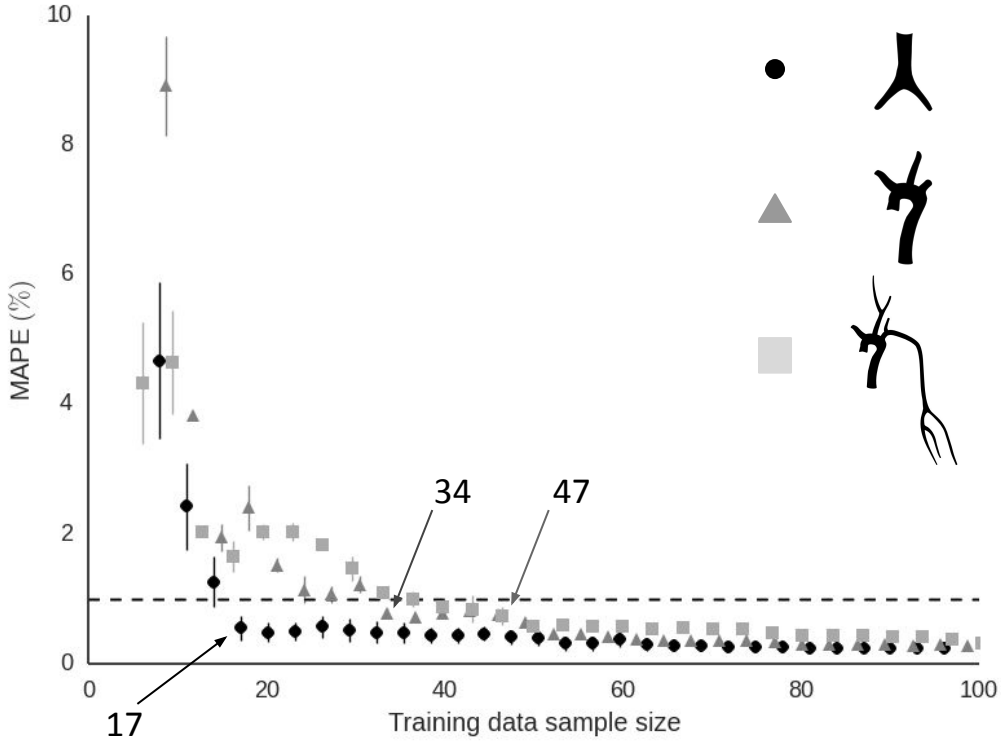


OR

Gaussian process emulator

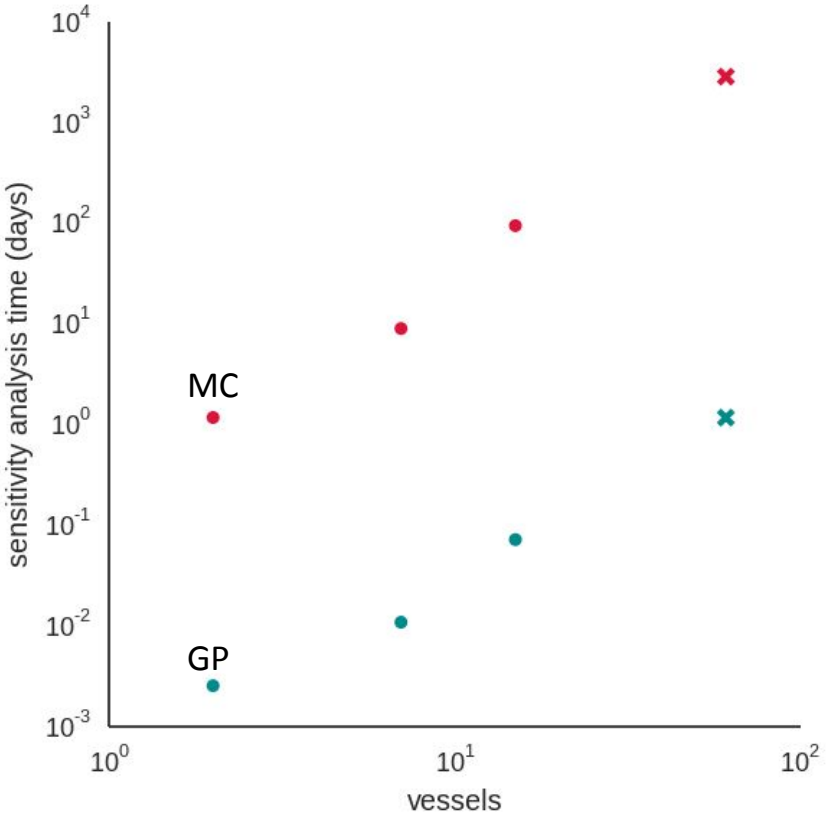


Prediction error

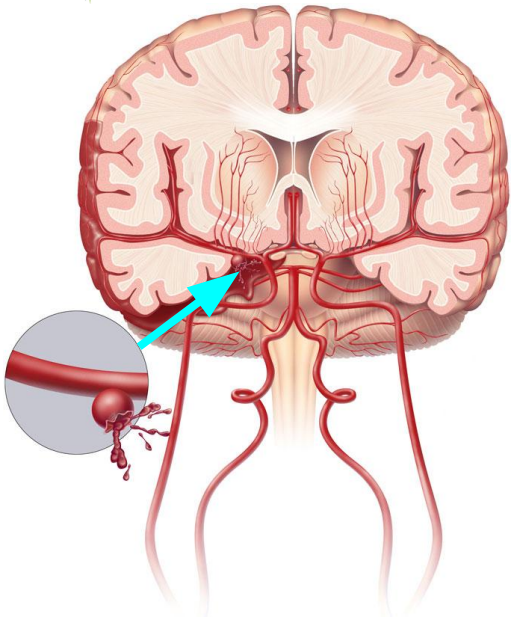


MC	GP
8000	17
29000	34
61000	47

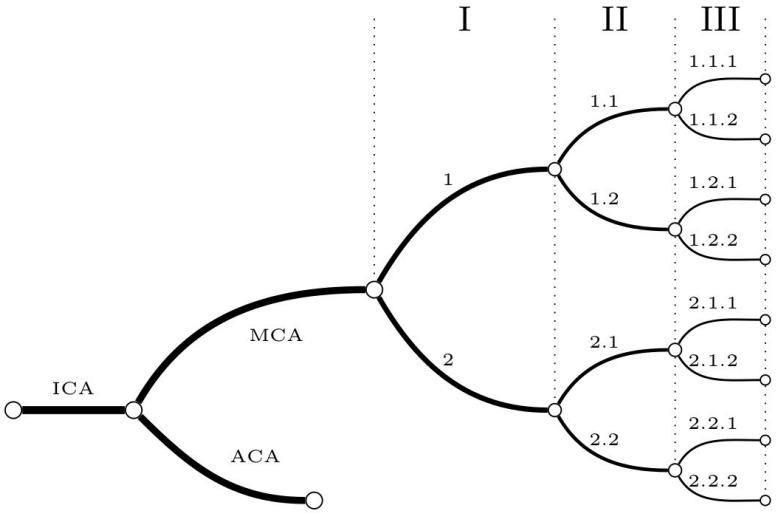
Running time



Cerebral vasospasm



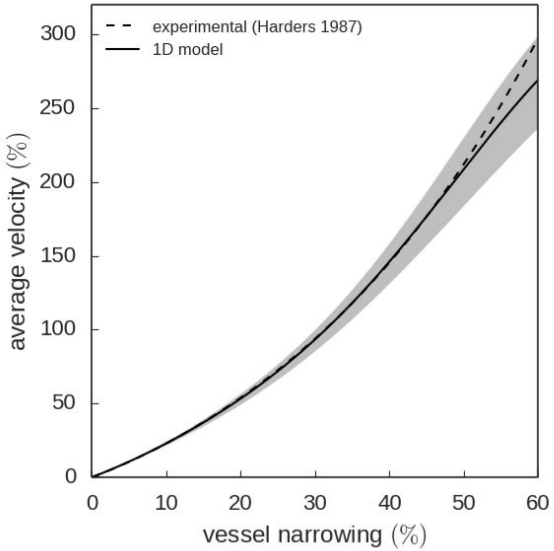
strokecenter.org

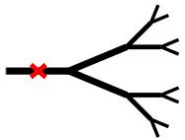


17 vessels

	R_0	E	ℓ	R_p	C_p
$\min(P)$	1	1	1	99	0
$\min(\partial_t P)$	90	7	6	0	2
$\min(\partial_{tt} P)$	94	6	4	0	0
$\min(Q)$	71	22	36	7	6
$\min(\partial_t Q)$	88	5	12	0	4
$\min(\partial_{tt} Q)$	73	7	25	1	5
$\min(u)$	97	0	2	0	0
$\min(\partial_t u)$	89	1	6	0	6
$\min(\partial_{tt} u)$	93	1	5	0	1
$\mathrm{mean}(P)$	3	0	1	98	0
$\mathrm{mean}(\partial_t P)$	50	33	46	9	5
$\mathrm{mean}(\partial_{tt} P)$	31	50	41	32	0
$\mathrm{mean}(Q)$	81	9	17	14	4
$\mathrm{mean}(\partial_t Q)$	59	54	54	0	1
$\mathrm{mean}(\partial_{tt} Q)$	65	35	65	0	1
$\mathrm{mean}(u)$	97	0	2	0	0
$\mathrm{mean}(\partial_t u)$	92	4	13	2	0
$\mathrm{mean}(\partial_{tt} u)$	88	16	35	0	2
$\max(P)$	18	1	2	81	0
$\max(\partial_t P)$	93	7	4	0	1
$\max(\partial_{tt} P)$	93	6	4	0	1
$\max(Q)$	92	3	16	4	1
$\max(\partial_t Q)$	86	2	17	3	2
$\max(\partial_{tt} Q)$	83	8	13	0	6
$\max(u)$	97	0	2	0	0
$\max(\partial_t u)$	97	1	2	0	0
$\max(\partial_{tt} u)$	88	3	8	0	5

Cerebral vasospasm





Conclusions

- Mechanistic modelling
- Sensitivity analysis
- Statistical emulator

- Application: cerebral vasospasm

Thank you

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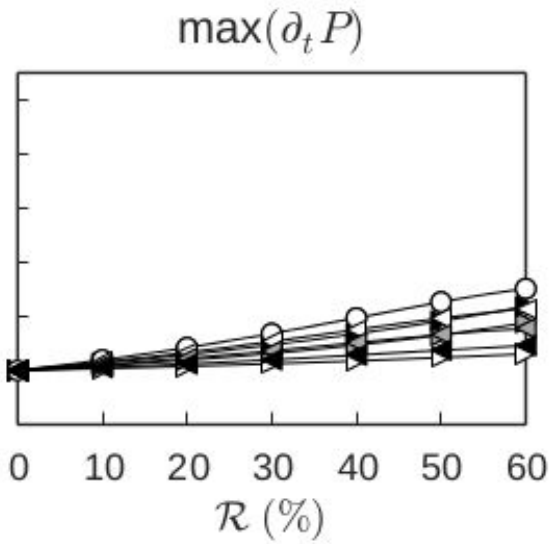
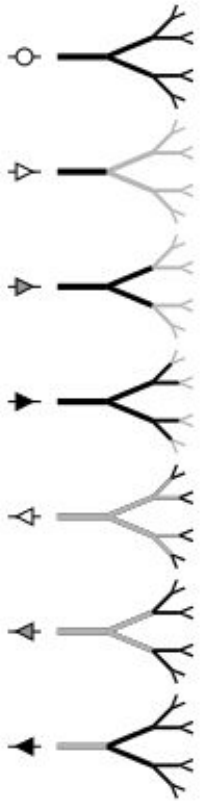

max($\partial_t P$)
0
10
20
30
40
50
60
$\mathcal{R}$ (%)



Models

#	v	o	d
1	2	1	8
2	7	4	29
3	15	8	61
4	61	31	245